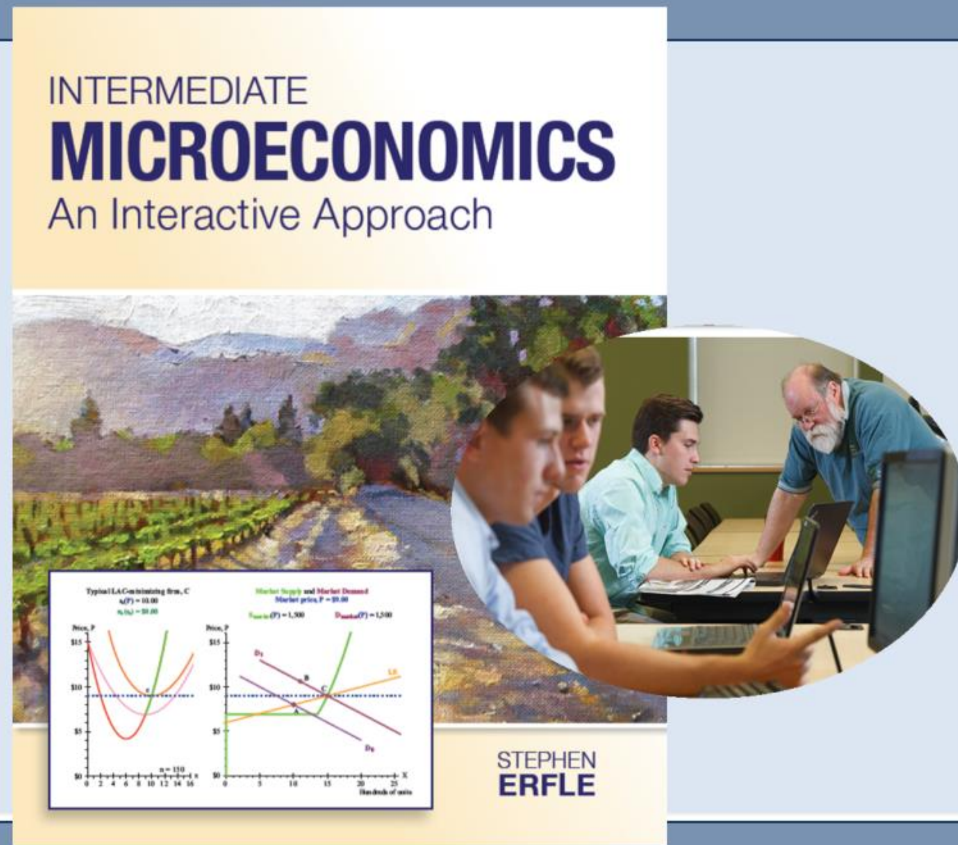
Stephen Erfle, PhD
Dickinson College

Digital Options start at

$39.95

Print + Digital Bundles

start at **$49.95**



3 key differentiators about the this new publication:

1. The overarching premise of this text…

is that microeconomics is most effectively learned in an active learning, interactive environment. Students have access to more than ***200 Interactive Excel Figures*** in the online text that allow them to move the graphs using sliders and click boxes. This interactivity helps students understand how graphic elements relate to one another. These files ***do not*** require knowledge of Excel.

2. More figures than are typical…

and many of the figures involve multiple scenarios of the same basic graph. Often the text employs interactive questions that require interpreting these scenarios; questions posed are answered at the bottom of the page.

3. Despite the geometric orientation…

this text is not light on algebraic analysis. The geometry is backed up by the relevant algebra. More than 500 equations are numbered for easy reference both within and across chapters. And, just like the geometry, the algebra is essentially error-free because it was used to create the graphs. The geometric orientation is perfect for the non-calculus enhanced classroom but the text can be readily used in a calculus-based class because a calculus treatment of the material is provided in appendices and endnotes, and calculus-based problems are included in the *Intermediate Microeconomics: An Interactive Approach Workbook*.

For Instructors

- ***Additional Interactive Excel Figure Files***, beyond those provided with the text. These files have built-in scenarios, not available on the student files, which show answers to Workbook questions.
- ***A Scenario Guide*** showing how to create and capture your own scenarios, so you can create exam questions using the interactive Excel files.
- ***Screenshots for Projection:*** Each chapter has a file with screenshots of all textbook figures together -- with the scenario Q&A's -- set up for overhead projection. The file is provided as a Word document, making it easy to hide the answers at the bottom of the screen for those figures with interactive questions.
- ***An Instructor Guide*** for each chapter provides a quick overview of materials covered in the chapter with teaching suggestions.
- ***The Workbook Answer Guide*** provides answers to each workbook problem as well as instructional hints regarding how to use workbook questions for additional instruction (and for test purposes).
- Multiple-choice questions that can be used to quiz understanding of the material.

- ***Direct Access to the Author*** for instructors with questions at: erfle@dickinson.edu

"For students" is on next page...

For Students

- ***The Interactive Excel Figure Files*** at the core of the text.
- ***End-of-chapter matching and short-answer questions*** with answers at the end of the text.
- **A Mathematical Appendix** offers help in three substantive areas:

  (1) 14-page review of algebra and geometry;
  (2) 13-page primer on basic derivative rules provided from a "user's perspective" for those wishing to incorporate calculus into the class;
  (3) 7-page primer on using Excel in economic modeling. Many of the topics are fleshed out using more than 20 extended examples drawn from portions of the text.

- ***The Student Guide*** provides a student perspective on the material. Written by three students and edited by the author. Provided electronically as a Word document so that students can add their own annotations as they read the text.
- **The Workbook** provides in-depth problems, a number of which utilize the interactive Excel files that are at the core of the text. These problems are designed to be turned in as homework.
- **All Offered at Affordable Prices…** along with the affordable/prices/media options for textbook.

Table of Contents/Preface/Sample Chapter follow...

Click here to request a Digital Review Copy
Click here to request a Paperback Copy

Click below for more on:

Contents

PART 2 CONSUMER THEORY

Preface

Anyone undertaking the job of writing an intermediate microeconomics text must first answer the question: Why write a new text when there are excellent texts already available? My answer has nothing to do with topic coverage, but rather with *how* those topics are covered. This text takes an active-learning, geometric approach to the subject by incorporating technology more centrally into the learning experience.

Static figures from the text are supplemented by dynamic counterparts that allow comparative statics analysis via mouse clicks. The interactive Excel files allow student exploration via sliders and click boxes to help students understand how graphic elements relate to one another. These files do not require knowledge of Excel.

The origin of this text can be directly traced to work I did more than 15 years ago, creating Excel materials for Edwin Mansfield's managerial economics text. Due to the success of those materials, Ed Parsons, then economics editor at W. W. Norton & Company, invited me to create materials for Joseph Stiglitz's introductory economics text in 2000. I jumped at this opportunity because I felt that Stiglitz would one day win the Nobel Prize. (He shared the Nobel with George Akerlof and Michael Spence [who taught me microeconomics and who was on my thesis committee while in graduate school] in 2001—you can read about their work in Chapter 21.) I wrote interactive tutorials in Word and Excel that were programmed into Flash. I later learned that those tutorials were the most accessed part of the web resources for Stiglitz's text.

Converting those materials into Flash was a challenge because graphic artists and programmers, unfamiliar with economic concepts, found it difficult to render graphic materials correctly, even when given accurate graphs to reproduce. (The average/marginal relation is perhaps the most common case in point. All economists understand how marginal pulls average, but graphic designers often do not understand this basic point, and inaccurate graphs result.) This experience led me to understand why so many economics texts have incorrect graphics. It also helped me to recognize that my comparative advantage lay in creating interactive graphic materials that could help students learn economics.

I began by working on electronic materials to support existing intermediate microeconomics textbooks for various publishers. But I soon came to believe that the best way to truly add value was to write my own text and to integrate the interactive material as the centerpiece. *Intermediate Microeconomics: An Interactive Approach* is the result.

The overarching premise of *Intermediate Microeconomics: An Interactive Approach* is that microeconomics is most effectively learned in an active-learning, interactive environment. One aspect of that interactivity is that students have access to more than 100 interactive Excel files, identified by the symbol in the text, that allow students to move the graphs using sliders and click boxes. Each of these graphs is "anatomically correct" and able to be moved via student interaction.

Additionally, *Intermediate Microeconomics: An Interactive Approach* has more figures than are typical, and many of the figures involve multiple scenarios of the same basic graph. The text often employs interactive questions that require interpreting these scenarios; questions posed are answered at the bottom of the page.

This geometric orientation does not mean that *Intermediate Microeconomics: An Interactive Approach* is light on algebraic analysis. The geometry is backed up with relevant algebra. More than 500 equations are numbered for easy reference both within and across chapters. And, just like the geometry, the algebra is essentially error-free because it was used to create the graphs. One benefit of this approach is that some of the more intricate or complex algebra (such as Cobb-Douglas cost in Chapters 10 and 11), which in other texts is provided in appendices with warning labels, can be more effectively examined because the essential structure of the algebra is readily visible in geometric relief.

Despite relegating calculus to appendices and endnotes only, *Intermediate Microeconomics: An Interactive Approach* readily could be used in a calculus-enhanced class. For example, Lagrangians are employed in the Chapter 6 Appendix, and lecture notes providing a geometric interpretation of Lagrange multipliers are provided in Chapter 6 of the *Instructor Guide*, which correlates with several Chapter 6 *Workbook* questions.

Flexible Text Organization

Most instructors spend the vast majority of an intermediate microeconomics course focused on the fundamentals and then pick and choose among applications as time permits at the end of the course. As a result, more time, energy, and ink are spent on the core material than on applications.

Intermediate Microeconomics: An Interactive Approach is composed of five parts. The core material is presented in Parts 2–4 (Chapters 3–16), and applications are provided in Part 5 (Chapters 17–22). Because some instructors may wish to explore some of the applications topics in a different order than is presented in the text, prerequisite information for each section is provided next to the section heading. Prerequisite information is also provided in Chapter 16 because some instructors will prefer to interleave the welfare discussion into the earlier parts of the text, rather than have a unified discussion after the various market structures have been discussed.

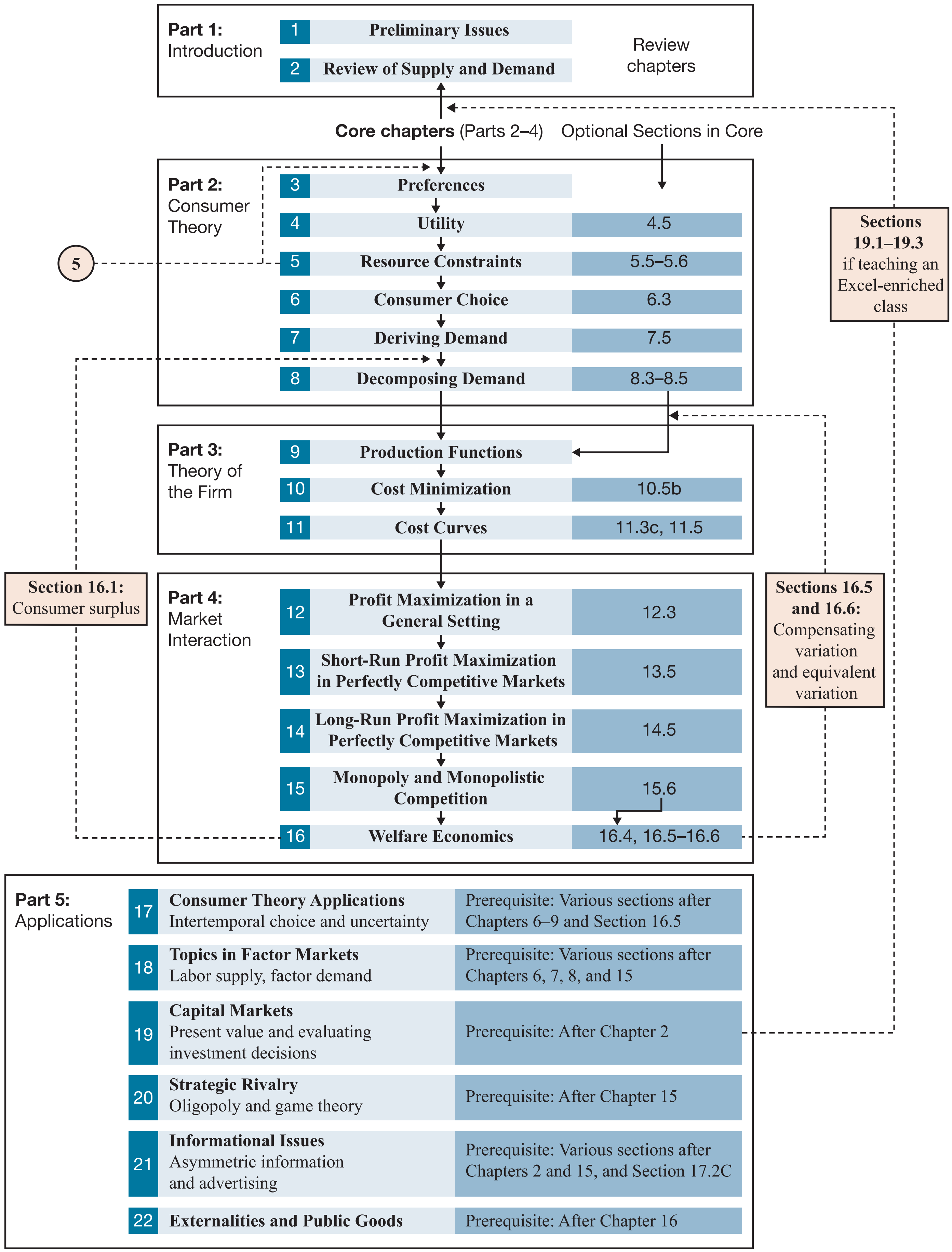
Even though Excel was heavily employed in writing *Intermediate Microeconomics: An Interactive Approach*, the text is written for traditional instructors who do not wish to teach an "Excel-enhanced" course. Those wishing to teach an Excel-enhanced course might want to bring the present value material from Chapter 19 to the front of the course. That material was written in a layered fashion, and it works quite well inserted in front of Chapter 3. See the schematic of the text denoting section dependencies. The schematic also delineates sections within the core that are optional.

By its very nature, *Intermediate Microeconomics: An Interactive Approach* is built for electronic delivery. Interaction with the Excel figure files that are at the core of the text is most seamlessly accomplished online. For those wishing to use a print textbook, however, one is available for purchase.

Special Features and Ancillaries for Students

Pedagogical and ancillary materials available for student use with this text include:

- The more than 100 interactive Excel files that comprise the core of the text.
- End-of-chapter matching and short-answer questions that provide basic problem-solving scenarios and a quick check on students' understanding of the material. Answers are provided in the Answers Appendix at the end of the text.
- A 34-page *Mathematical Appendix*, available online. This appendix covers three substantive areas: (1) a 14-page review of algebra and geometry, (2) a 13-page primer on basic derivative rules provided from a "user's perspective" for those wishing to incorporate calculus into the class, and (3) a 7-page primer on using Excel in economic modeling. Many of the topics are fleshed out using more than 20 extended examples drawn from portions of the text.
- The *Student Guide to Intermediate Microeconomics: An Interactive Approach*, available online. This guide offers the views of three students *who worked through the text material themselves*. It provides insights regarding where the students got stuck and how they got "unstuck" in the learning process. The *Student Guide* is



Schematic of Text Organization, Denoting Section Dependencies and Optional Sections

Solid arrows describe proposed flow; dashed arrows suggest alternative flow paths.

provided as a Word document so that students can add their own annotations as they read the text.

- The *Intermediate Microeconomics: An Interactive Approach Workbook*, which provides in-depth problems, a number of which utilize the interactive Excel files that are at the core of the text. These problems are designed to be turned in as homework. Sometimes, the problems build off earlier problems. Problems that are likely to be turned in on different days are placed on different pages. For the same reason, the *Workbook* is delivered single-sided, ready for a three-ring binder.

Instructor Supplements

Materials available for instructor use include:

- Additional interactive Excel files, beyond those provided with the text. These files have built-in scenarios, not available in the student files, which show answers to *Workbook* questions.
- A guide that shows instructors how to create and capture their own scenarios so that they can generate exam questions using the interactive Excel files.
- For each chapter, a Word file that provides screenshots of all textbook figures, together with the scenario Q&As, set up for overhead projection—an exceptionally useful tool when teaching from the text. These chapter files are provided as Word documents because they can be easily manipulated in class, for example, to hide the answers at the bottom of the screen to figures with interactive questions.
- An *Instructor Guide* that provides a quick overview of materials covered in each chapter, along with teaching suggestions.
- The *Workbook Answer Guide*, which provides answers to each *Workbook* question, as well as suggestions regarding how to use *Workbook* questions for additional instruction (and for test purposes).
- Each chapter also has a set of multiple-choice questions for quizzing students' understanding of the material.

Acknowledgments

I am indebted to a number of people who helped bring this project to market. First and foremost are Ed Laube and Tom Doran of Textbook Media Press, who were willing to take on a project of this scope. Mary Monner and Victoria Putman, of Putman Productions, LLC, were a delight to work with on editorial issues; this text is certainly better because of their efforts.

I also wish to thank Philip Young, who in 2012 invited me to join him in coauthoring a new edition of an existing managerial economics text. Keat, Young, and Erfle, *Managerial Economics: Economic Tools for Today's Decision Makers*, 7th edition, Pearson Education, 2013, was the end product of this collaboration. That experience and Phil's encouragement regarding the materials I had created reignited my interest in completing work on this text.

Part of those coauthor responsibilities was to write a chapter on game theory and asymmetric information. I based that chapter on material I had written for this text early on. Rather than rewrite those sections, I asked two former Dickinson College students, now economists in their own right, if they would like to write those sections of this text. I wish to thank J. Kerry Waller of Piedmont University for the game theory portion of Chapter 20 and Jue Wang of the University of California, San Diego, for the asymmetric information portion of Chapter 21.

I have benefited from comments and critiques from a number of Dickinson College students over the course of this project. I wish to single out for thanks Susan Cerniglia and Holly Kirkpatrick, who provided research assistance early in the writing process, and

John Mayers, who was my research assistant this past year. He, together with David Erfle and Jacob Milligan, wrote the *Student Guide* mentioned earlier.

Although I have used many of the *Workbook* questions over the years in intermediate microeconomics and managerial economics classes, I want to single out for thanks the group of 12 students who participated in a semester-long workshop during Spring 2006. These students worked through the text, as well as beta versions of the *Workbook* questions, in a workshop format.

Finally, I want to thank my family. My children—Dave, Kate, and Vera—certainly saw less of me during this process, and it was often in front of a computer screen. My wife, Laura, held it all together and, in addition, offered a keen eye for detail as she read various versions of the manuscript. I wish to thank all of them for helping me see this project through to the end.

About the Author

Stephen Erfle
Photo courtesy of Carl Socolow, Dickinson College.

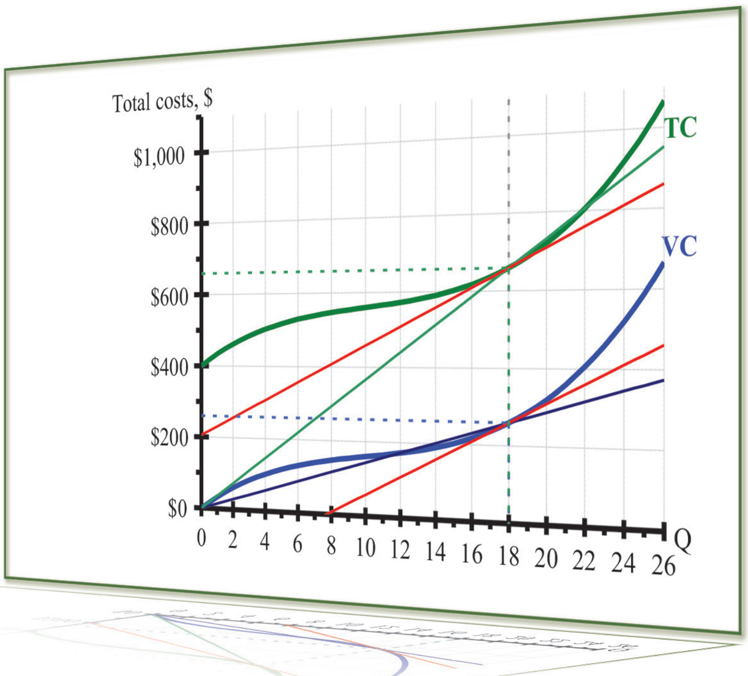
Dr. Stephen E. Erfle received a BS in mathematics and a BA in economics from the University of California, Davis, and an MA and PhD in economics from Harvard University. During the late 1990s, Dr. Erfle helped found the International Business and Management Department and major at Dickinson College. By 2012, this had become the largest major at Dickinson College. He has also taught in the Economics Department at Dickinson College and in the School of Social Sciences at the University of California, Irvine.

Trained as a microeconomic theorist, Dr. Erfle is an interdisciplinary scholar with published research in a variety of fields, ranging from communications theory, political geography, economics, and economics pedagogy, to the psychology of sports and public health. He is coauthor with Philip Young and Paul Keat of *Managerial Economics: Economic Tools for Today's Decision Makers*, 7th edition, Pearson Education, 2013. His interest in managerial economics stems from a 14-month sabbatical at Seagram Classics Wine Company in the mid-1990s, during which he maintained offices at Sterling Vineyards and at Mumm Cuvée Napa, where the finance and marketing departments of SCWC resided. This sabbatical also provided his introduction to Excel and solidified in his mind the need to integrate Excel into a business curriculum, due to its ubiquity in the business world. As noted in the Preface, that experience also eventually led to this text.

Dr. Erfle has fielded questions from instructors regarding his managerial teaching materials for a number of years, and he is happy to continue that practice with this project. He can be reached at erfle@dickinson.edu.

Cost Curves

CHAPTER 11



Chapter Outline

The cost analysis in Chapter 10 links the cost of factors of production with how the factors are combined to produce goods. Cost minimization is based on a comparison of the relative value of factors of production in producing the good and the relative market valuation of those factors. The more direct analysis based on isoquants and isocost lines that forms the core of Chapter 10 is indirectly used in this chapter, where we examine cost as a function of output. The cost curves developed in this chapter form the basis for profit-maximizing decisions by the firm.

Cost is used in a variety of settings and in a variety of disciplines, so it is important to delineate how the economist's notion of cost differs from others. To an economist, opportunity cost is of preeminent importance. Other cost concepts examined in Section 11.1 include accounting cost; implicit and explicit cost; fixed, variable, and total cost; and sunk cost. Section 11.2 examines the geometry of short-run cost functions. Costs can be analyzed on both a total and a per-unit basis, just as production functions can be analyzed on a total and a per-unit (of input) basis. Of the seven cost functions examined, three are total, and four are per unit. The notion of "average versus marginal" that first surfaced in the discussion of average versus marginal productivity in Chapter 9 reemerges in this chapter. Section 11.3 shifts the analysis from short-run to long-run cost. The relation between short-run cost and long-run cost that was first examined in Section 10.5 is reexamined in the context of cost as a function of output. Section 11.4 examines the algebra that ties together cubic cost functions; cubic cost functions are the simplest functions that depict the changing productivity we often observe in production processes. The chapter concludes with a second analysis of cost of production, given Cobb-Douglas technology. The Cobb-Douglas total cost results derived in Chapter 10 form the basis for the long- and short-run cost functions presented in Section 11.5.

11.1 Various Notions of Cost

opportunity cost: The opportunity cost of a resource is the value of that resource in its next best alternative use.

To an economist, the overarching concept that must be applied when considering all types of cost is the notion of opportunity cost. The **opportunity cost** of a resource is the value of that resource in its next best alternative use. Another term used for opportunity cost is *economic cost*. Resources are scarce and can only be applied to one task at a time. Suppose you have two part-time jobs during the summer. If you work an hour at the fast-food restaurant, you cannot work that same hour at the convenience store. The opportunity cost of each job is the value forgone by not being able to do the other one. Similarly, the opportunity cost of deciding to hang out with friends for an hour when you could have worked is the hour's wages you gave up by hanging out with your friends. Consider a second example: Suppose you own both halves of a duplex; you live on one side, and you rent out the other. Suppose that the current tenant decides to leave, and your mother-in-law moves into the vacant unit so she can save on rent and be closer to the grandkids. The opportunity cost of this move is the rent forgone by not being able to rent to other renters once your mother-in-law moves in. (Indeed, in this event, the opportunity cost is much, much higher, but you would need to be "Married with Children" to really understand!)

For many of the resources used in the production process, the market price of that input acts as its opportunity cost. Recall from our discussion at the start of Section 10.1 that this price is a rental rate per unit of time. For labor, this price is a wage rate; for capital, it is the rental rate on a unit of capital; and for land, it is the rental rate per unit of land. Raw materials prices are the price per unit of the raw material.

explicit costs: Explicit costs are out-of-pocket expenses.

implicit costs: Implicit costs are costs that are not explicitly paid out but are nonetheless incurred.

Not all costs, however, have a price tag attached to them. It is important to distinguish between explicit and implicit costs. **Explicit costs** are out-of-pocket expenses. Wages paid, raw material purchases, and rent paid on property that is being leased are examples of explicit costs. Explicit costs tend to be easier to quantify than implicit costs. **Implicit costs** are costs that are not explicitly paid out but are nonetheless incurred. Implicit costs are inherent in the notion of opportunity cost. Suppose you give up a $40K-a-year job to start your own company. At the end of the year, you are ecstatic because you have broken even and indeed had an accounting net income of $25K for the year. An economist would not look at this as making an economic profit because it ignores the opportunity cost of your own labor in determining net income. From an economist's point of view, the accounting profit of $25K in net income must be balanced against what you lost by giving up your job; this is the value of your labor in its next best alternative use. An economist would restate this as the accounting profit of $25K less the implicit cost of $40K, or an economic loss of $15K due to the opportunity cost of your labor. Since the foregone wages are only implicit in this instance, they are somewhat harder to quantify. This does not mean that they should be ignored.

Consider once again starting your own company. Suppose you use your own savings to help fund this start-up. If these funds would have earned you $5K in income had you invested elsewhere (with another start-up operation or a money market fund, for example), then the $5K in foregone income is another implicit cost that should be considered when determining the profitability of the start-up.

accounting costs: Accounting costs are the costs reported by accountants on financial reports.

These examples of explicit versus implicit costs point to one difference between an accountant's and an economist's perspective. Interestingly, while accountants and economists both discuss "cost," they have different definitions of what comprises cost. As a broad brushstroke, accountants view costs in a historical context, while economists use the notion of opportunity cost to examine the current context. **Accounting costs** are the costs reported by accountants on financial reports. These costs include actual expenses and depreciation, following established rules (in the United States, these rules are called the Generally Accepted Accounting Principles [GAAP]). Financial accountants describe past performance for external audiences—most notably, investors—in quarterly and annual reports, and for tax purposes.

Suppose you are an appliance manufacturer who uses an integrated circuit chip as part of the production process. Three months ago, you purchased 5,000 units at $10 per unit, and you are working through this inventory as you produce appliances. You find that you can now purchase the same chip for $8 per unit, due to changes that have occurred in the integrated chip market. How should you value your remaining inventory of chips in this instance? Accountants would use the historic price of $10 as their point of reference; economists would use the $8 current price as their point of reference. Economists determine value by using the next best alternative use for the factor, and if that factor can now be bought or sold for $8 per unit, that is more relevant than what the factor originally cost. Managerial accountants attempt to bridge the gap between the financial accountant's need to provide reliable information for external audiences and the business manager's need to obtain reliable information for making decisions.

Opportunity costs comprise total cost and can be further classified according to the short-run/long-run distinction described in Section 10.5 as either fixed cost or variable cost. **Fixed cost (FC)** is the opportunity cost that accrues to factors of production that are fixed in the short run. **Variable cost (VC)** is the opportunity cost that accrues to factors of production that are allowed to vary in the short run. **Total cost (TC)** is the sum of fixed and variable cost. Since fixed cost accrues to factors that do not change in the short run, they are themselves fixed. More explicitly stated: Fixed cost does not vary as output produced varies. By contrast, variable cost varies as output varies, since output changes by adjusting the usage of variable factors of production.

fixed cost (FC): Fixed cost is the opportunity cost that accrues to factors of production that are fixed in the short run.

variable cost (VC): Variable cost is the opportunity cost that accrues to factors of production that are allowed to vary in the short run.

total cost (TC): Total cost is the sum of fixed and variable cost.

One type of cost is not included in an economist's decision calculus. If a cost has already occurred but its opportunity cost is zero, then it should not be included, even though it is a readily visible explicit cost. Economists call such costs sunk costs. A **sunk cost** is a cost that has already occurred and cannot be recovered. The defining feature of a sunk cost is the inability to recover the cost; the next best alternative use has zero value. An important example of a sunk cost is an option to purchase a factor of production (as long as the option does not allow resale). Consider the following example, which examines options for crude oil.

sunk cost: A sunk cost is a cost that has already occurred and cannot be recovered.

Suppose you are a crude-oil refiner, and you wish to reduce the risk you face in the crude-oil market because of rising prices. One solution is to purchase an options contract to purchase crude oil at a fixed price in the future. An **options contract** gives the purchaser the ability to purchase a product at a given date in the future for a fixed price. This price is called the *exercise price* or *strike price*. The price of an option depends on the time remaining before the option expires, the exercise price, the volatility in the price of the product being traded, and interest rates.[1] Consider the following scenario:

options contract: An options contract gives the purchaser the ability to purchase a product at a given date in the future for a fixed price.

1. The current open-market price of crude oil is $40/bbl (per barrel).
2. An options contract to purchase crude at an exercise price of $40/bbl 3 months from now costs $1.50/bbl.

The total price per barrel in this instance would be $41.50/bbl. Assume you purchase an options contract for 1,000 barrels at a cost of $1,500. Three months from now, the open-market price of crude is $41/bbl. You wish to purchase 1,000 barrels of oil. Should you exercise your contract or let it expire? The answer in this instance is you should exercise your contract and purchase the 1,000 barrels at $40/bbl, rather than purchase the barrels on the open market at $41/bbl. The appropriate comparison ignores the sunk cost of $1,500, since this cost is not recoverable. The decision is not to purchase 1,000 barrels of oil at $41,500 or $41,000, but to purchase at $40,000 or $41,000. The $1,500 has already been spent (regardless of whether you exercise the option or let it expire) and should therefore have no effect on the current decision. In this instance, you should exercise the contract whenever the open-market price is higher than the exercise price, and you should let it expire when the reverse holds true.

The same logic applies to other types of options as well. When firms are considering where to locate a plant, they often purchase an option on a piece of land. The option provides them the right to purchase the land at a given price, as long as the sale occurs within a fixed period of time. If another piece of land becomes available prior to exercising the option, the cost of the option should have no bearing on the choice between the two pieces of property. The option cost has already occurred and is not recoverable. Of course, if the decision being considered is whether or not to purchase the option, then the option cost has *not* already occurred, and it should be considered as part of the decision-making process. The truisms "Let bygones be bygones" and the business version "Don't throw good money after bad" attempt to get at this point.

A final important concept necessary to understand costs relates to the units used to discuss them. Broadly speaking, costs can be described in two ways—total cost and per-unit cost. To take a simple example, suppose it costs $1,500 to produce 1,000 two-by-fours. This can be described as $1,500 in total cost and as a per-unit cost of $1.50. Both of these methodologies have their uses. The total cost version tends to provide the greatest intuition, at least initially. The per-unit cost version is more useful for decision making. Both can be viewed from either a short-run or long-run perspective.

Opening a Take-Out Pizza Shop

To put the cost notions just discussed in perspective, consider the decision to open a take-out pizza shop near a college campus. Suppose that Steve is considering giving up his $24K-per-year job as an assistant manager of a retail store to open a take-out pizza shop. He faces various decisions regarding how he should proceed and these decisions relate to various factors of production:

- *Location*—A storefront suitable for the shop is currently available. Steve has talked to the owner of the property, and the owner is offering an option to rent the property on a month-to-month basis for $1,200 per month. This option is good for 30 days and costs $700.
- *Equipment*—A local restaurant supply company is willing to lease pizza ovens, refrigeration units, preparation tables, and miscellaneous utensils to Steve. This equipment rents for $800 per month, with a one-time setup fee of $1,000.
- *Utilities*—Steve estimates that telephone, gas, and electric will cost $300 per month. The hookup fees for these utilities are an additional $200.
- *Advertising*—The local newspaper has a special deal for new businesses that provides $1,000 worth of advertising per month (based on the paper's normal rates) for $500 per month. To qualify, you must agree to purchase the advertising every month that you are open in the first year. The shop's phone number is listed in the Yellow Pages as part of the monthly telephone fee, but a quarter-page Yellow Pages advertisement costs $600. This lump-sum fee occurs on an annual basis when the new edition of the Yellow Pages is published.
- *Ingredients*—Steve estimates that cheese, toppings, tomato sauce, spices, and flour will cost $2.50 per pizza.
- *Packaging*—Generic pizza boxes are available from a cardboard fabricator for $1 per box. These boxes can be purchased in any quantity. Personalized boxes (with logo, address, and phone number) are available for $1.50 per box but must be purchased in minimum lots of 5,000 boxes.
- *Labor*—Steve plans to run the shop himself, and he plans to pay himself out of the profits from the business. He figures that he will need to hire a part-time worker for peak hours on Friday and Saturday evenings. The part-time worker would cost him $400 per month.

Suppose that Steve moves forward with his plan by taking out the option on the storefront and pays rent. He obtains the equipment and utilities, and takes advantage of the special newspaper advertising deal, as well as the quarter-page advertisement in the Yellow Pages. He decides to use the generic pizza boxes, hires the part-time worker, and begins making pizzas. How should he view each of the costs he has incurred? Are they fixed, variable, or sunk? Implicit or explicit? How would your answer change if Steve had made other choices for advertising and packaging? (Try to work through each part before reading the analysis in the next few paragraphs.)

Steve has incurred a variety of costs in this instance. Consider first which of the costs are fixed and which are variable. All of the costs except ingredients and packaging are fixed in nature because each is independent of the number of pizzas made. The ovens must be on and rent must be paid, whether one or a thousand pizzas are made. Some of these costs, however, are fixed, while others are sunk. The $700 option to rent the storefront is the most obvious sunk cost. Once it is made, it should have no bearing on further decisions because it has already occurred, and it is not recoverable. Similarly, the $1,000 equipment setup fee, the $200 utilities hookup fee, and the $600 Yellow Pages advertising fee are sunk costs because their opportunity cost is zero—each has no alternative use. If Steve decides to go out of business, he cannot recover any of these fees. The phone company may allow a business that has been in place for some time and is stable to spread out the $600 cost of the advertisement over 12 months at $50 per month. In this case, the $600 would not be sunk at the beginning of the year, and the Yellow Pages advertisement would be considered a fixed—not sunk—cost.

The packaging is a variable cost: It varies as output varies. The same would not be true, at least as a startup, had Steve decided to use the personalized boxes instead. It is true that the price quoted is a per-unit price, but the minimum lot size is 5,000 boxes. Suppose that Steve had decided to pursue this course of action, instead of buying boxes in smaller lots as needed. The purchase would have cost Steve $7,500, and if Steve went out of business at a point when he had 3,000 boxes left, the remaining boxes would have no recovery cost (other than for recycling purposes). The problem in this instance is that the personalized boxes cannot be resold or used by another pizza shop; the value of the boxes in their next best alternative use is zero (or at most, the salvage value of the cardboard). The personalized boxes are more expensive on several scores: They cost more on a per-unit basis, they must be purchased in larger lot sizes, *and* they have no resale value. (If you look around at independent pizza shops near your college, it should not be surprising that most use generic boxes.) As we have seen, much of the cost involved in producing takeout pizza is fixed in nature, and the only variable costs of production in this instance are $3.50 per pizza on a per-unit basis ($2.50 for ingredients and $1.00 for packaging).

All costs are explicit save one in this instance. Steve planned to run the shop himself and pay himself out of the profits from the business. His labor is an implicit cost because his labor services have a next best alternative use. Steve gave up a $24K-per-year job to undertake the start-up. An economist would include in the cost calculation Steve's implicit labor cost of $2,000 per month when determining the costs inherent in running the shop.

11.2 The Geometry of Short-Run Cost Functions

Short-run cost functions can be viewed from two orientations: total cost and per-unit cost. (Each orientation requires its own diagram, but the diagrams are tied to each other, just as total product and marginal product requires two diagrams that are tied to each other via a common horizontal axis.) Short-run total costs are comprised of two components—fixed cost and variable cost:

$$TC = FC + VC. \qquad \textbf{(11.1a)}$$

Before we discuss these cost functions, it is worth quickly mentioning their shorthand references. The complete name for *total cost* is *short-run total cost (STC)*, *fixed cost* is *short-run total fixed cost (SFC)*, and the complete name for *variable cost* is *short-run* *total variable cost (SVC)*. The "short run" is omitted unless we are discussing both short-run and long-run costs together (and any discussion of long-run costs will explicitly state that the costs are long run). Similarly, each of the per-unit cost functions will have prefaces (average or marginal) attached to their name that signify that the cost function being examined is a per-unit cost function. Unless otherwise stated, we can omit both prefaces and assume that variable cost is short-run total variable cost and fixed cost is short-run total fixed cost. We begin with the total cost orientation, since it ties directly to the production and cost-minimization discussion of Chapters 9 and 10.

Total Costs

Variable cost varies as output varies, and therefore, variable cost may be written as VC(Q). When there are multiple variable input factors, the contribution of each should be included in determining variable cost. As discussed in Section 10.5, these variable factors should be used in a cost-minimizing fashion to produce a given level of output—that is, production should be on the short-run expansion path (SREP). In the take-out pizza example in Section 11.1, production was Leontief in ingredients and packaging.[2] A pizza requires one "unit" of ingredients and one box; ingredients cost \$2.50/pizza, and packaging is \$1.00/pizza, so $VC(Q) = \$3.50 \cdot Q$.

In the two-input (L, K) model, variable costs are labor costs, and labor costs vary, depending on how much output we wish to produce and the wage rate. Figure 11.1A describes a total product curve, $Q = TP(L)$. For reasons described in Section 9.2, labor initially has increasing marginal productivity, but past the point of inflection, it exhibits decreasing marginal productivity. The variable cost of producing Q units of output is:

$$VC(Q) = w \cdot L(Q). \quad \textbf{(11.1b)}$$

L(Q) is the amount of labor necessary to produce Q units of output, given K_0 units of capital. L(Q) is the inverse of the total product curve. The second line beneath the horizontal axis in Figure 11.1A simply takes L and multiplies by w; in this instance, w = \$100 (think of this as a daily wage rate). These numbers are variable cost. Variable cost as a function of output, VC(Q) in Figure 11.1B, is simply the total product curve rotated 90° with the vertical axis relabeled as variable cost in dollars, rather than units of labor. (Because w is denominated in dollars per unit of labor and L is denominated in units of labor, $(\$/L) \cdot L = \$$, $w \cdot L = VC$ is measured in dollars.)

The variable cost curve in Figure 11.1B is based on a specific wage rate; a different wage rate leads to a different variable cost curve. The total product curve in Figure 11.1A is fixed because (1) it describes technological possibilities that are not affected by factor prices, and (2) capital is fixed. The Excel file for Figure 11.1 allows you to vary the wage rate from \$1 to \$100 in \$1 increments to see this change dynamically. While the position of the cost curve in Figure 11.1B changes as the wage rate changes, its shape does not change. The two graphs are always direct inverses so that, for example, the output level where the point of inflection occurs remains the same in both Figures 11.1A and 11.1B (at Q = 10). Only the units on the vertical axis of the variable cost graph change as w changes. For example, if w = \$10, then the vertical axis labels in Figure 11.1B are in increments of \$10, rather than \$100, and if w = \$1, then the vertical axis labels are in increments of \$1.

The changing convexity of the variable cost curve is a direct result of the changing convexity of the total product curve on which it is based. Although the horizontal axis in Figure 11.1A is related to the vertical axis in Figure 11.1B (via $VC = w \cdot L$), the easiest way to talk about both graphs is to talk in terms of quantity (the vertical axis in Figure 11.1A and the horizontal axis in Figure 11.1B). For Q between 0 and 10, the TP curve is convex upward, and consequently, the VC curve is convex downward over the same range. For Q larger than 10, the reverse holds true: TP is convex downward, and consequently, VC is convex upward.

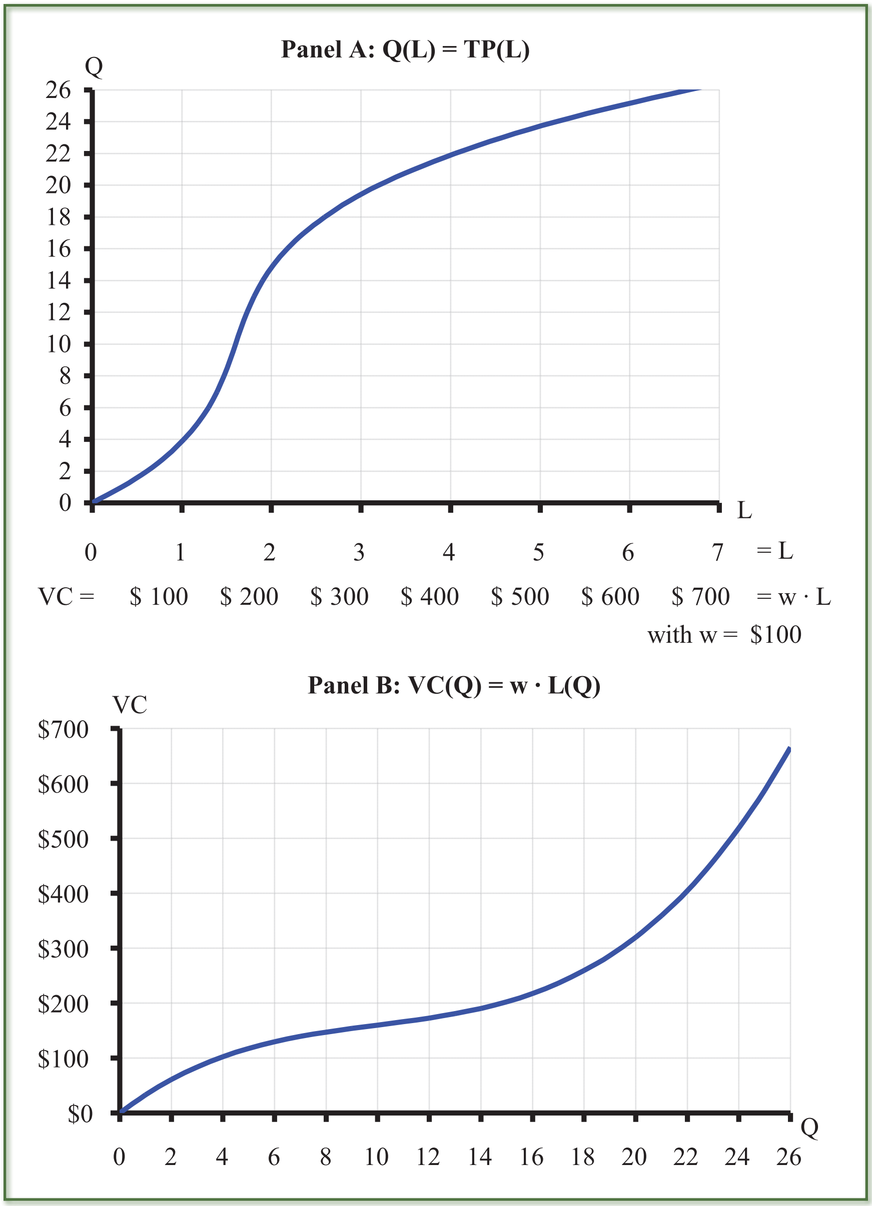


FIGURE 11.1 Deriving Short-Run Variable Cost from Short-Run Total Product

Q1a: How would either Figures 11.1A or 11.1B change if w = \$10 rather than \$100?

Q1b: Would a change in wage alter the quantity where the point of inflection occurs in Figure 11.1B?*

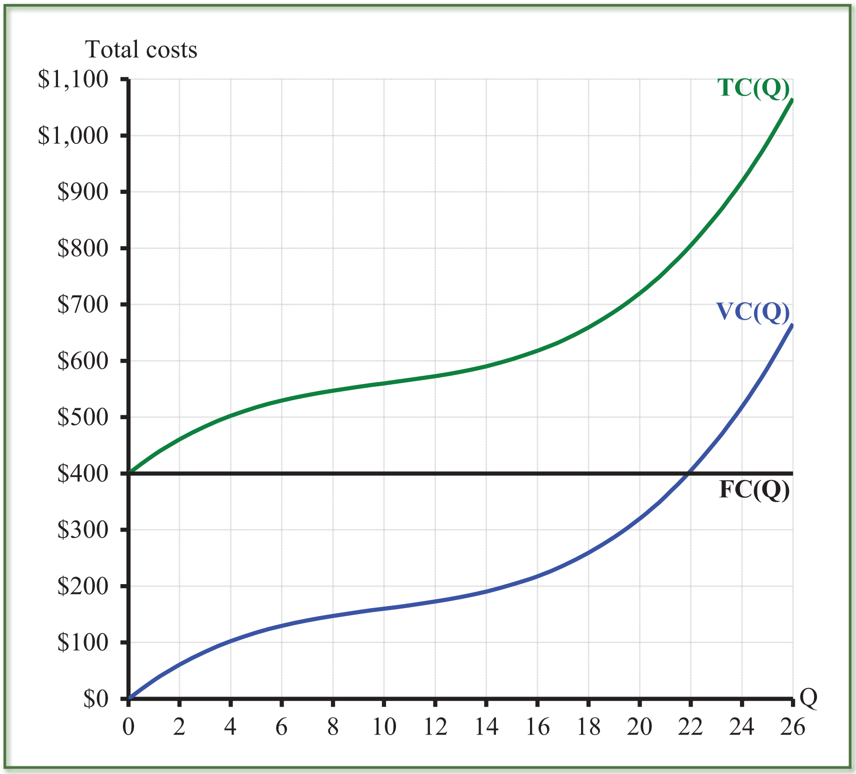
When working with multiple fixed components in the short run, total fixed cost is simply the sum of the individual fixed costs. In the pizza example, the monthly fixed cost is the sum of rent, equipment rental, utilities, newspaper advertising, and labor.

Q. What is the monthly total fixed cost in the pizza example from Section 11.1?†

***Q1 answer:** There would be no change to Figure 11.1A because a change in factor prices would not affect short-run production. Figure 11.1B would look the same if you reduce each number on the vertical axis by a power of 10 (i.e., the range for VC is from \$0 to \$70). The quantity of the point of inflection would not change because it is determined by the output level having the maximum MP_L in Figure 11.1A.

†Q answer: Total monthly fixed cost is \$5,200 = \$1,200 + \$800 + \$300 + \$500 + \$400 + \$2,000. The most difficult of these costs to see is the \$2,000 opportunity cost of Steve's time.

FIGURE 11.2 Short-Run Total Cost Curves



In the two-input (L, K) model, fixed costs per unit of time are:

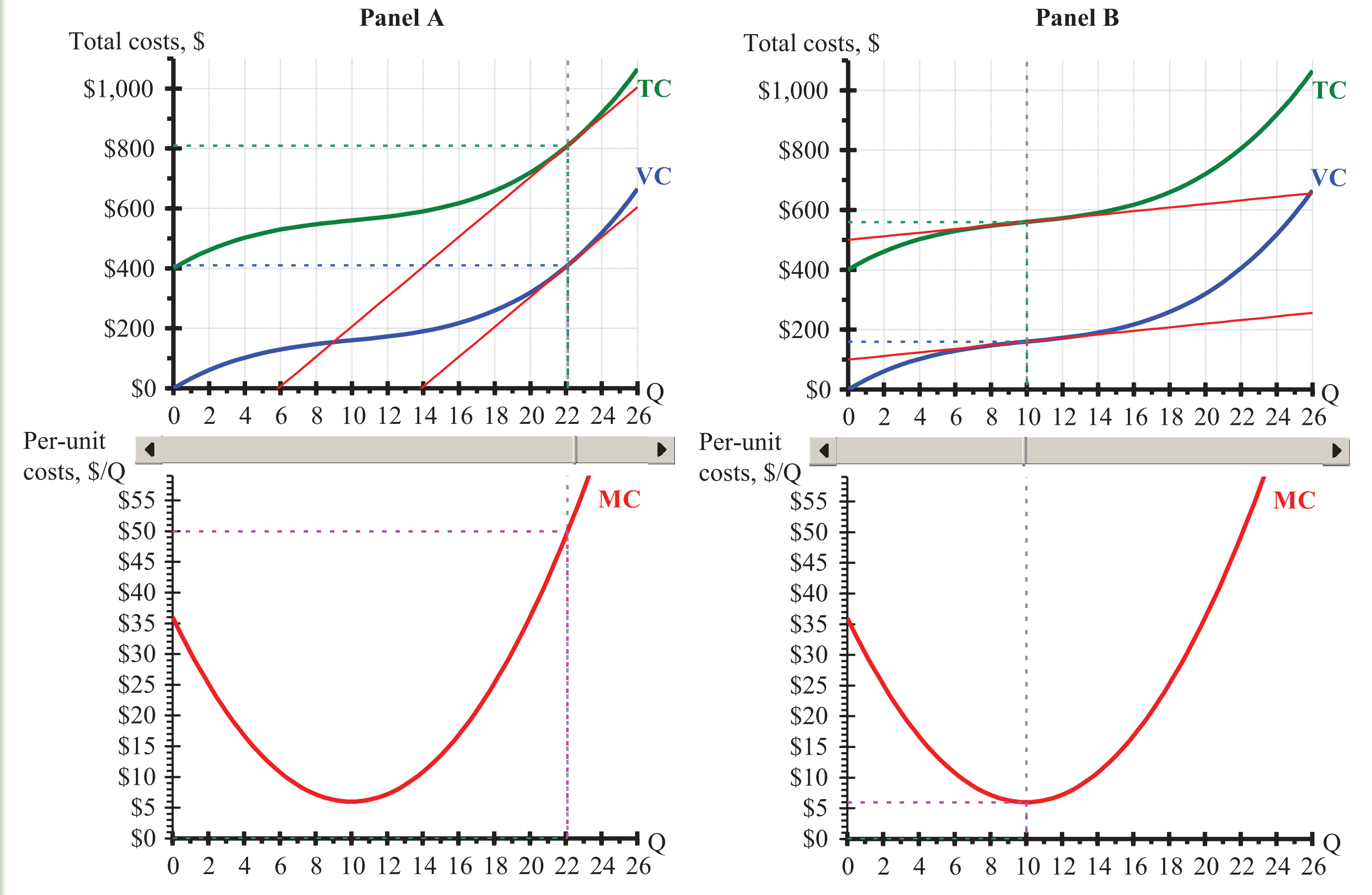
$$FC(Q) = r \cdot K_0. \qquad \textbf{(11.1c)}$$

The rental rate on capital is r, and the firm employs K_0 units of capital in the short run. Even though fixed cost is formally written as a function of quantity, its fixed nature makes it, in reality, independent of quantity. This cost is incurred regardless of the number of units of output produced. The only way to avoid this cost in the short run is to go out of business. Fixed cost is the horizontal line at a height of $FC(Q) = r \cdot K_0 = \$400$ in Figure 11.2. Once we have VC(Q) and FC(Q), TC(Q) is easily seen as their sum, according to Equation 11.1a. The green short-run total cost curve, TC(Q), is simply the variable cost curve translated up by fixed cost.

Per-Unit Costs

marginal cost (MC): Marginal cost is the incremental cost of producing one more unit of output.

The per-unit counterparts to the total cost curves just discussed are more useful for decision-making purposes. The most important of these curves is marginal cost. **Marginal cost (MC)** is the incremental cost of producing one more unit of output. This is more formally described as a *short-run marginal cost (SMC) curve*, but as discussed earlier, we usually drop the short-run S. As with other "marginal" concepts in the text, this is calculated as a slope, but the question is: the slope of what? In this instance, there are two equally valid answers: Marginal cost is the slope of the variable cost curve and the slope of the total cost curve. Incremental cost can be measured using either curve. This is not surprising in light of the relation that holds between variable cost and total cost described in Equation 11.1a and Figure 11.2; these two curves differ by a constant amount (FC). Algebraically, $MC(Q) = \Delta TC(Q)/\Delta Q = \Delta VC(Q)/\Delta Q$ for small changes in quantity, ΔQ.[3] Marginal cost is shown at two levels of output in Figure 11.3. Both Figures 11.3A and 11.3B depict two diagrams tied by a common horizontal axis. The upper diagram in each reproduces the TC(Q) and VC(Q) curves from Figure 11.2 and adds a red line tangent to both curves at a given output level. The first thing to note about both figures is that the tangent lines are parallel; this will always be the case, since TC(Q) is simply a vertical translate of VC(Q).[4]



slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
22.1	$ 410	$ 810	$49.92	$ -	$ -

slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
10.0	$ 160	$ 560	$ 6.00	$ -	$ -

FIGURE 11.3 Deriving Marginal Cost from Variable Cost or Total Cost

The output level in Figure 11.3A was chosen so that the slope could be calculated using the grid system in the upper diagram. Verify that marginal cost is approximately $50 per unit by calculating the slope of either of the red tangent lines. Figure 11.3B depicts the point where marginal cost achieves its minimum. As described in Figure 11.1, Q = 10 is the point of minimum marginal cost *and* the point of maximum marginal product of labor. Of course, if we had chosen a different level of capital, then the short-run production function would change, and the output level that achieves maximum marginal product of labor and minimum marginal cost would also change. A larger level of capital (i.e., a short run with $K_1 > K_0$) would typically be associated with a total product curve in which a larger amount of labor is required before labor achieves its maximum productivity and, hence, a larger amount of output before achieving minimum marginal cost.

This tie between labor productivity and marginal cost can be seen in Figure 11.1. For output levels below Q = 10 (to the left of the vertical line at Q = 10 in Figure 11.1B), marginal cost is decreasing, since the marginal product of labor is increasing (below the horizontal line at Q = 10 in Figure 11.1A), and for output levels above Q =10, marginal cost is increasing because the marginal product of labor is decreasing. The output level associated with maximum marginal productivity is also the minimum marginal cost level of output because both are associated with the point of inflection that occurs at the same output level for both curves (as argued previously).

average total cost (ATC): Average total cost is total cost divided by quantity, ATC(Q) = TC(Q)/Q.

average variable cost (AVC): Average variable cost is variable cost divided by quantity, AVC(Q) = VC(Q)/Q.

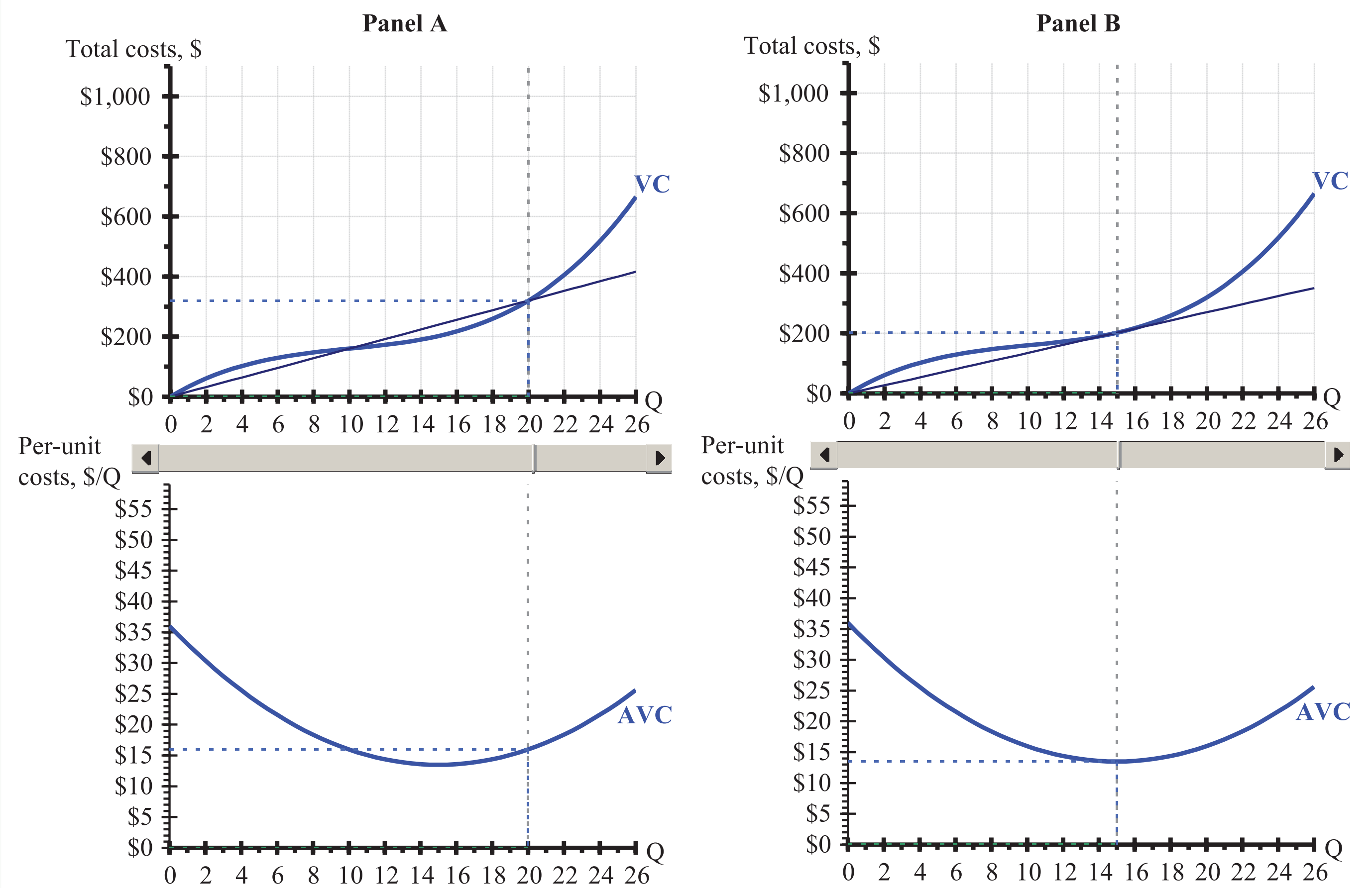
average fixed cost (AFC): Average fixed cost is fixed cost divided by quantity, AFC(Q) = FC(Q)/Q.

In addition to marginal cost, the other three per-unit curves are "average" versions of each of the three total cost curves, TC(Q), VC(Q), and FC(Q). As discussed earlier, the more formal preface S for "short-run" is implicit in each definition. **Average total cost (ATC)** is total cost divided by quantity, ATC(Q) = TC(Q)/Q. **Average variable cost (AVC)** is variable cost divided by quantity, AVC(Q) = VC(Q)/Q. **Average fixed cost (AFC)** is fixed cost divided by quantity, AFC(Q) = FC(Q)/Q. The relation between these three curves can be seen algebraically by dividing both sides of Equation 11.1a (TC = FC + VC) by Q to obtain:

$$ATC(Q) = AFC(Q) + AVC(Q). \qquad \textbf{(11.2)}$$

We use the same geometric trick to examine the concept of average cost as we used to examine average productivity in Figure 9.2. *The slope of the chord connecting the origin to any point on a cost curve is the average cost for that level of output.*

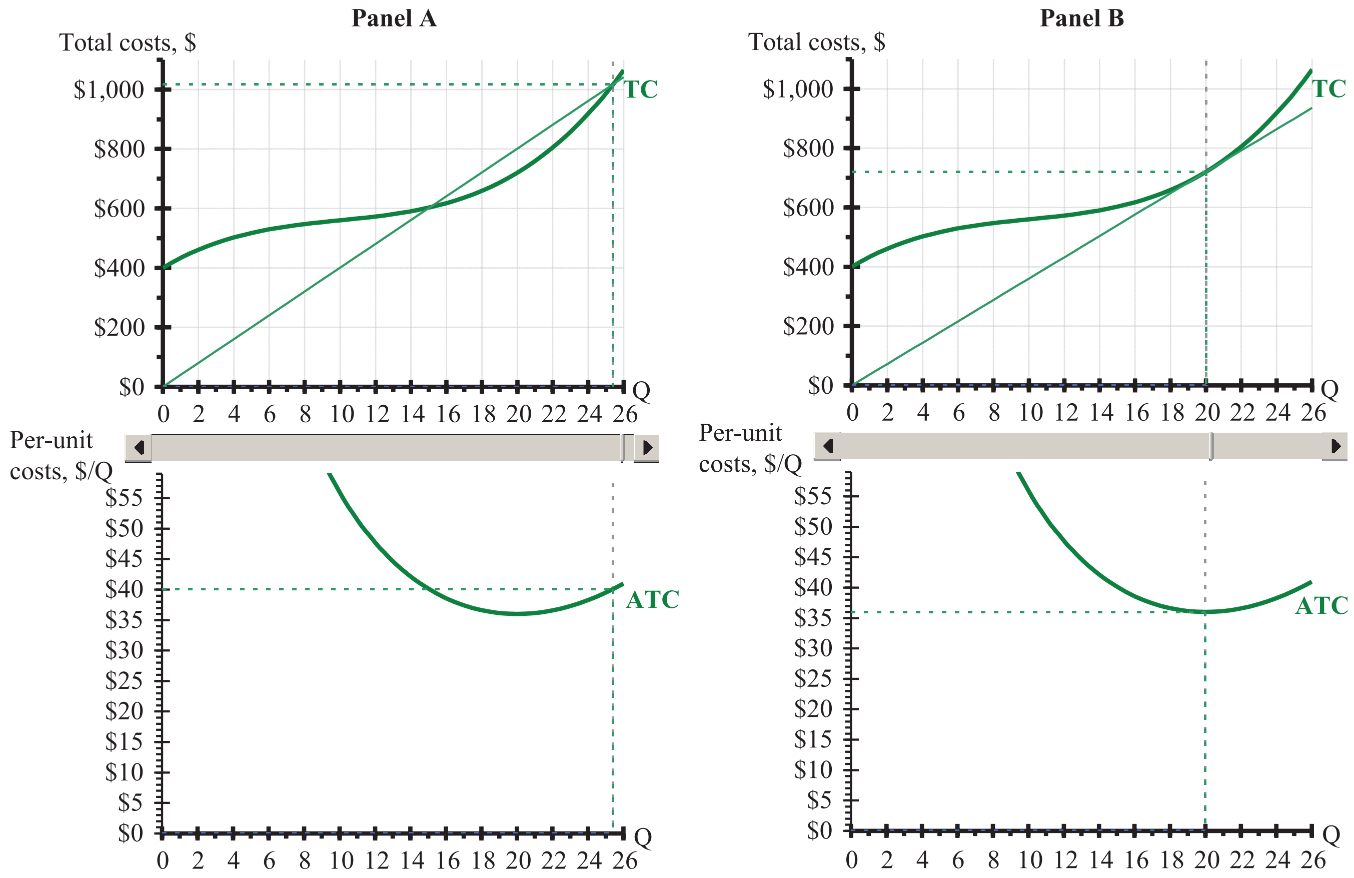
Average variable cost is the slope of the chord connecting the origin to points on the variable cost curve in Figure 11.4. The blue chord in the upper diagram of Figure 11.4A has a slope of 16 (it passes through the point (25, \$400); the slope of this chord is 16 = 400/25). The blue chord passes through the variable cost curve at Q = 10 and Q = 20; this corresponds to AVC(10) = \$16 and AVC(20) = \$16 in the lower diagram. The point of minimum average variable cost is found by finding the chord that is tangent to the VC(Q) curve. This occurs at approximately (15, \$200) in the upper diagram. (In Chapter 9, you



slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
20.0	\$ 320	\$ -	\$ -	\$16.00	\$ -

slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
15.0	\$ 203	\$ -	\$ -	\$13.50	\$ -

FIGURE 11.4 Deriving Average Variable Cost from Variable Cost



slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
25.4	$ -	$1,018	$ -	$ -	$40.06

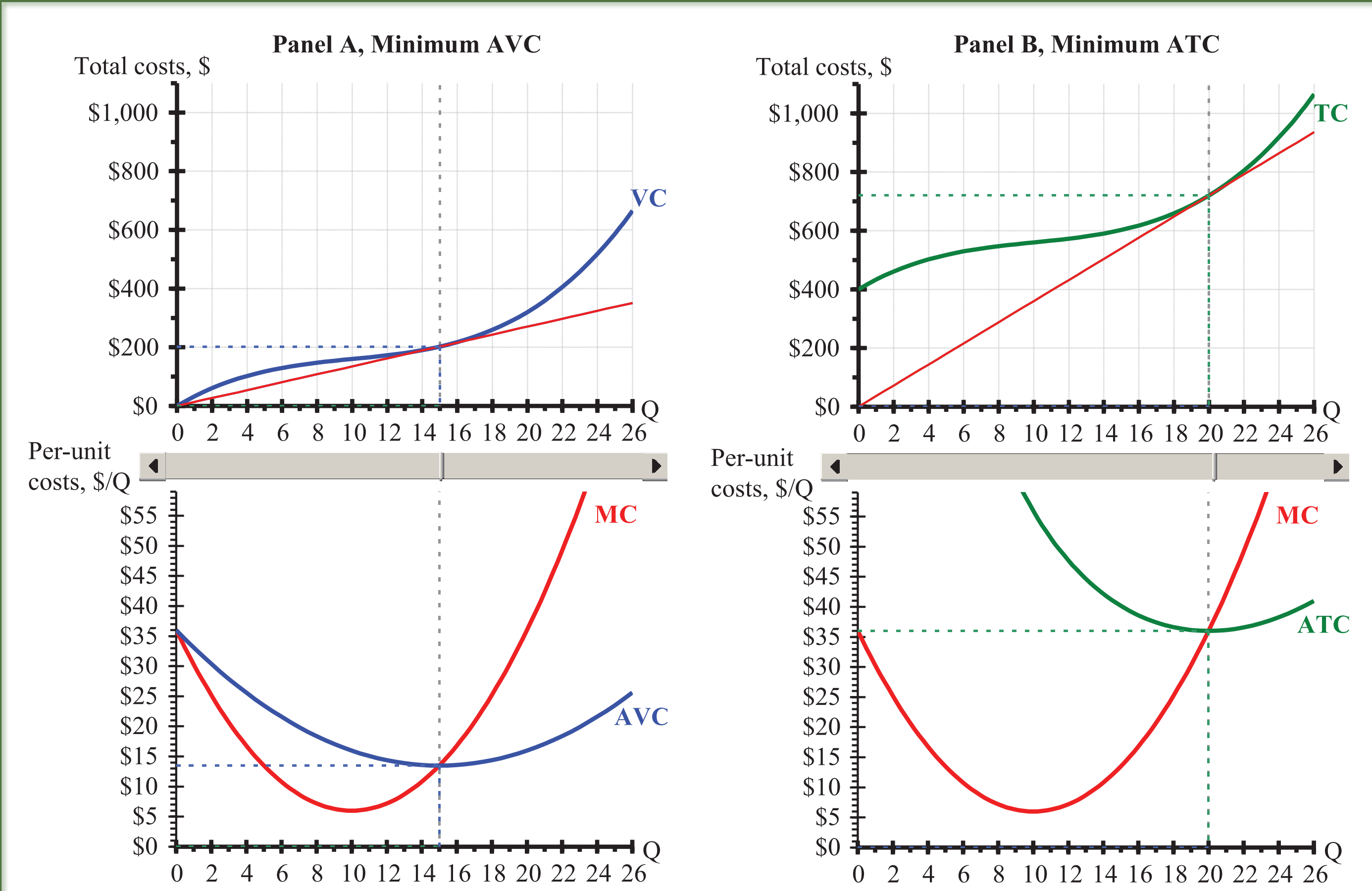
slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
20.0	$ -	$ 720	$ -	$ -	$36.00

FIGURE 11.5 Deriving Average Total Cost from Total Cost

were asked to think of the horizontal axis as the hinge of a hardcover book. The minimum occurs at the output level where you have just opened the book far enough that you “kiss” the curve [at Q = 15].) Not surprisingly, Q = 15 is also the point of maximum average product of labor in Figure 11.1A.

Average total cost is the slope of the chord connecting the origin to points on the total cost curve in Figure 11.5. The green chord in the upper diagram of Figure 11.5A has slope 40 (40 = 800/20), and it intersects the total cost curve at about Q = 25.4 and Q = 15. These output levels correspond to the two points on the ATC(Q) curve in the lower diagram, where ATC(Q) = $40. Figure 11.5B shows the point of minimum average total cost at (20, $720) in the upper diagram. This is the point of tangency between the total cost curve and the chord that just “kisses” the TC(Q) curve. In the lower diagram, this is the point of minimum average total cost; this occurs at Q = 20 and ATC(20) = $36 ($36 = $720/20).

In addition to these associations between the TC and ATC curves and between the VC and AVC curves, there is also a relation between marginal and average. Figure 11.6 examines the points of minimum average variable and average total cost. These are also points for which marginal equals average. The common theme in both Figures 11.6A and 11.6B is that at this output level, the slope of the line tangent to the curve (marginal) equals the slope of the chord (average).This is shown in the upper diagrams in Figures 11.6A and 11.6B, which resemble the upper diagrams in Figures 11.4B and 11.5B, except that the blue and green chords and red tangent line overlap, and consequently, the chord cannot be seen. If you use the Excel file for Figure 11.6 and adjust Q, the average chord disappears



slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
15.0	$ 203	$ -	$13.50	$13.50	$ -

slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
20.0	$ -	$ 720	$36.00	$ -	$36.00

FIGURE 11.6 Comparing Marginal Cost to Average Cost

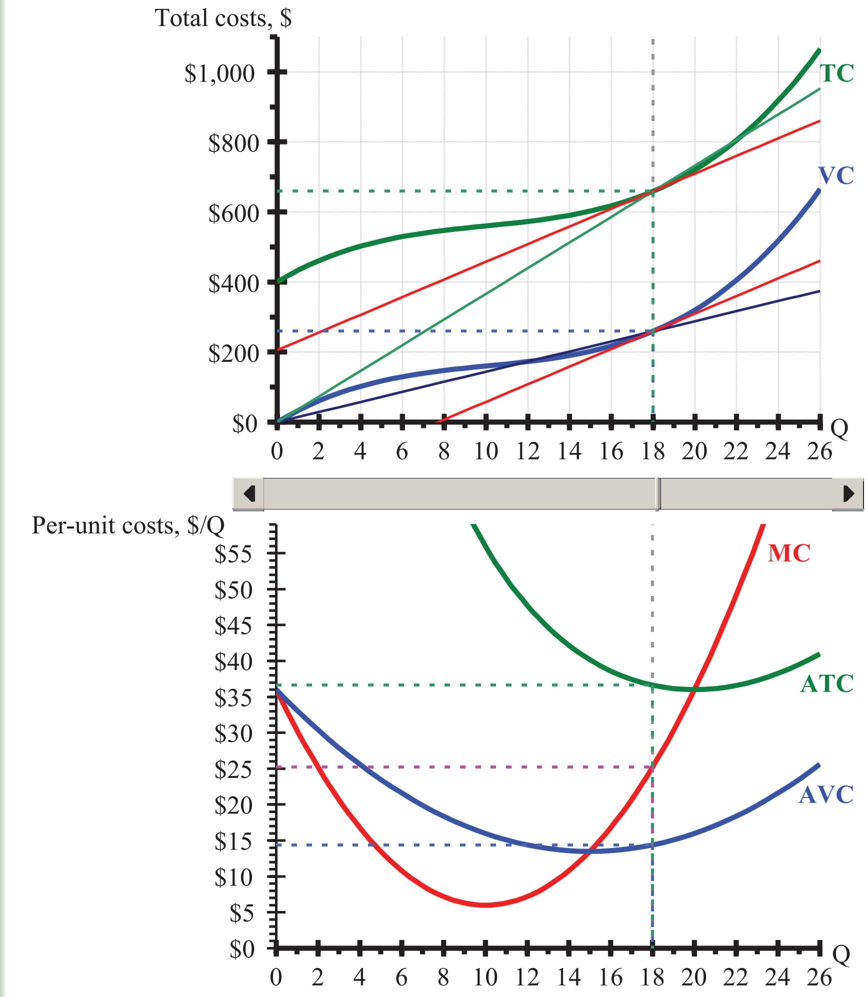
and reappears as you move away from these output levels. *Average always equals marginal at the minimum of average, regardless of whether it is average variable or average total cost.* Chapter 9 outlined a similar relation between average and marginal productivity; we can tell whether average is increasing or decreasing at a given output level by finding out where marginal is in relation to average. The same rule applies here as in Chapter 9:

> **Claim:** *Marginal pulls average up or down, depending on whether it is above or below average.*

(If this still feels uncomfortable, reread the discussion in Section 9.2 regarding batting averages and GPAs.)

Figure 11.7 combines all of the elements described in Figures 11.3–11.6. An output level was chosen, Q = 18, such that ATC > MC > AVC. This ordering is clear in the lower diagram of Figure 11.7 (and in the accompanying table), but it is worthwhile to examine the slopes of the chords and tangents to TC(18) and VC(18) so that we can verify the ordering of per-unit costs. The red tangents (MC) are parallel (as required because TC is a vertical translate of VC). At this output level, the tangent is steeper than the blue VC(18) chord but flatter than the green TC(18) chord. This is geometric proof that AVC(18) < MC(18) < ATC(18).

For simplicity, one per-unit cost function has been omitted thus far in our discussion. Average fixed cost is FC/Q, or by regrouping Equation 11.2, AFC(Q) = ATC(Q) – AVC(Q). This is shown for Q = 10 in Figure 11.8, with total fixed cost equal to $400. AFC(Q) in



slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
18.0	$ 259	$ 659	$25.20	$14.40	$36.62

FIGURE 11.7
Comparing AVC, ATC, and MC

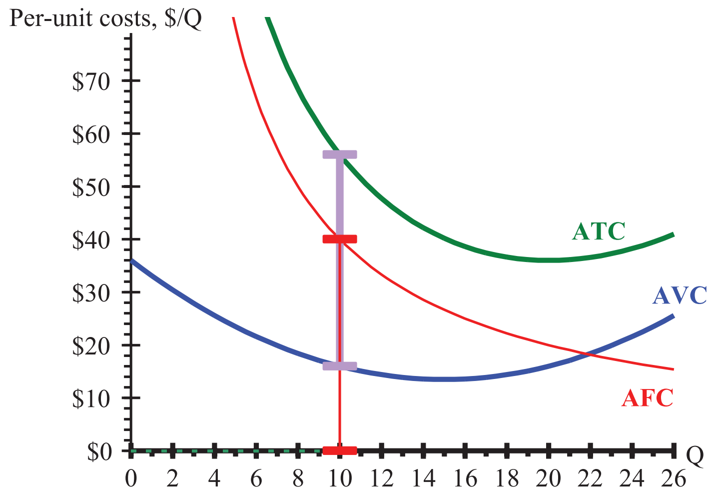
Figure 11.8 can be derived in two ways. It is most directly the red rectangular hyperbola (AFC = FC/Q), but it is also the difference between ATC and AVC at any quantity level. The vertical red line segment from the Q axis to AFC and the vertical lavender line segment from AVC to ATC are the same length for any Q (and that length is FC/Q). Given Q = 10, both line segments are $40 long. When Q = 20, both are $20, and so on. In effect, the AFC curve provides no new information (since it is already included in earlier per-unit figures as ATC – AVC). As a result, economists often omit AFC from graphs of per-unit cost curves.

Before leaving the discussion of short-run cost functions, we must review what is being held constant in the short run. Most obviously, fixed costs are held constant, but these costs are constant because fixed factors of production are held constant in the short run. Indeed, we defined the short run as the time frame in which some factors of production are held fixed. And, of course, technology is held fixed in the short run; a given bundle of inputs produces a certain amount of output, and this does not change in the short run.

Short-run cost curves are based on fixed-factor prices. If these prices change in the short run, then the short-run cost curves will shift. The easiest way to see this is to imagine all factor prices doubling. This would double the cost of producing any level of output because fixed cost and variable cost both double in this instance. Since capital is fixed,

FIGURE 11.8 Average Fixed Cost as AFC(Q)= ATC(Q) – AVC(Q)

Therefore, economists typically do not show AFC.



slider Q	VC(Q)	TC(Q)	MC(Q)	AVC(Q)	ATC(Q)
10.0	$ 160	$ 560	$ -	$16.00	$56.00

the short-run expansion path does not change. The only change is the cost of achieving that production level. Suppose instead that only variable factor prices double, but that fixed-factor prices do not change. How would this change short-run cost curves? Consider each of the three total cost curves and each of the four per-unit curves, and try to answer what will happen to each before reading the answer in the next paragraph.

Variable cost and average variable cost would double for all output levels. This can be seen using the Excel file for Figure 11.1; change w = $50 to w = $100 and the variable costs in Figure 11.1B double for the fixed total product curve in Figure 11.1A. Since the price and quantity of fixed factors do not change, neither does fixed cost or average fixed cost. Because variable cost doubles, marginal cost doubles as well (since marginal cost is the slope of variable cost, and doubling variable cost doubles the slope of the VC curve at any point). Total cost and average total cost increase, but they do not double (since fixed cost remains fixed).

11.3 Long-Run Cost Curves

As we discussed in Chapter 10, if the firm is able to adjust all factors of production, it is able to produce output at lower cost than if it cannot. We saw this directly in the comparison of the long-run expansion path (LREP) with the short-run expansion path (SREP) in Figure 10.10. The cost curves examined in Section 11.2 are based on the short-run expansion path: Short-run cost curves hold capital fixed (or more generally, all fixed factors are fixed). Different levels of capital lead to different short-run cost curves, so each short-run cost curve is based on a fixed capital stock. We now turn to an analysis of cost curves that are appropriate to long-run decision making.

long-run total cost curve, LTC(Q): The long-run total cost curve, LTC(Q), is the curve relating the total cost of production to the level of output produced when all factors of production are allowed to vary.

long-run average cost curve, LAC(Q): The long-run average cost curve, LAC(Q), is the curve relating the average cost of production to the level of output produced when all factors of production are allowed to vary.

In the long run, there are no fixed factors of production; therefore, there are no fixed costs in the long run, and *all* costs are variable. This reduces the number of cost curves that need to be considered to three: (1) long-run total cost (LTC), (2) long-run average cost (LAC), and (3) long-run marginal cost (LMC). The **long-run total cost curve, LTC(Q)**, is the curve relating the total cost of production to the level of output produced when all factors of production are allowed to vary. The LTC curve is the representation in (Q, $) space of the cost associated with points on the LREP (just as STC(Q) in Section 11.2 is the representation in (Q, $) space of the total cost associated with points on the SREP). The **long-run average cost curve, LAC(Q)**, is the curve relating the average cost of production to the level of output produced when all factors of production are allowed to vary. The **long-run marginal cost curve, LMC(Q)**, is the curve relating the incremental cost of production to the level of output produced when all factors of production are allowed to vary. Note that we do not have to call LAC by the longer name LATC, since there are no fixed costs, and therefore, there is nothing to distinguish between average total cost and average variable cost.

long-run marginal cost curve, LMC(Q): The long-run marginal cost curve, LMC(Q), is the curve relating the incremental cost of production to the level of output produced when all factors of production are allowed to vary.

We initially focus on LTC and LAC, since both are based on short-run total cost curves and average total cost curves in a common fashion. Both are envelopes of their respective short-run curves. For every quantity, there are multiple short-run total cost curves. For each output level, one of these curves provides the least-expensive option. An **envelope** is a curve drawn by connecting these least-cost points for all quantities. An envelope is most easily seen geometrically.

envelope: An envelope is a curve drawn by connecting least-cost points for all quantities.

LTC and LAC: The Geometry of Envelopes

Suppose that there are only two plant sizes available: small and large. The small plant produces a smaller amount of goods cheaper than a large plant, and vice versa. This is the situation depicted in Figure 11.9. Figure 11.9A provides the total cost analysis, while Figure 11.9B provides the per-unit cost analysis. The large plant is the short-run cost function that formed the basis for Section 11.2 (note in particular that FC_{large} = \$400 in Figure 11.9A), while the small plant produces output using much less capital (although we cannot determine this with precision from the graph, in this instance, FC_{small} = \$32). When output is less than about 8.5, the small plant produces output more cheaply than the large plant. You would choose the small plant if you expect to produce fewer than 8.5 units, and

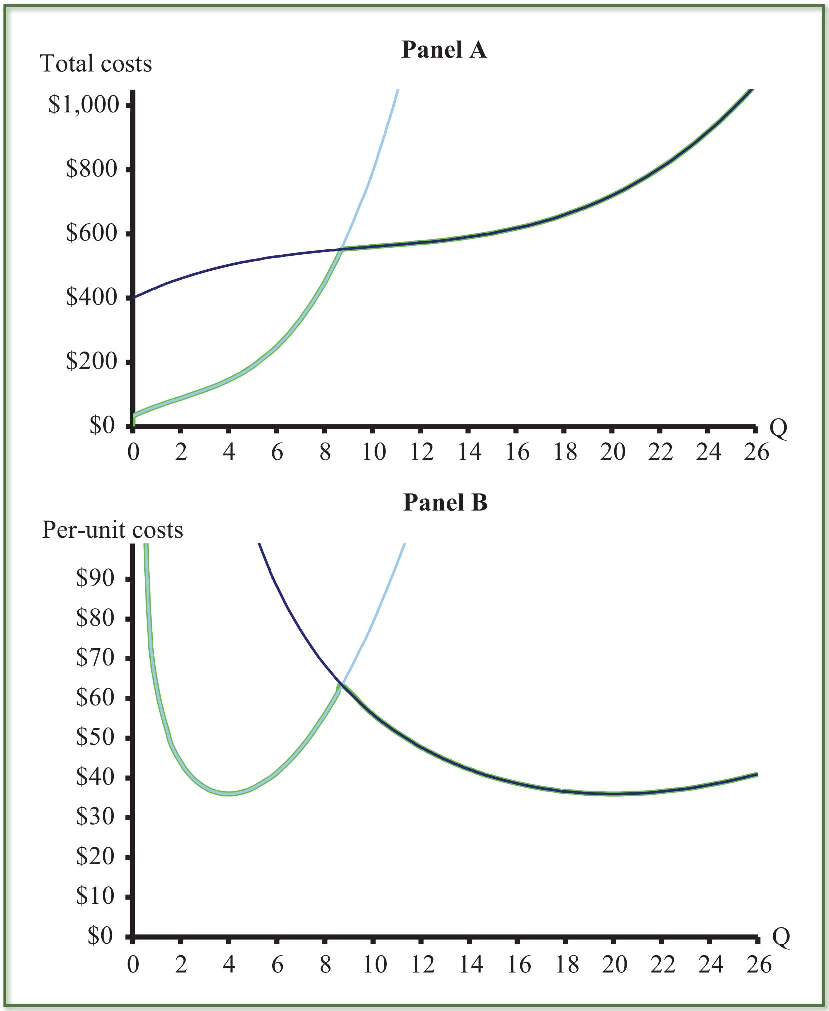


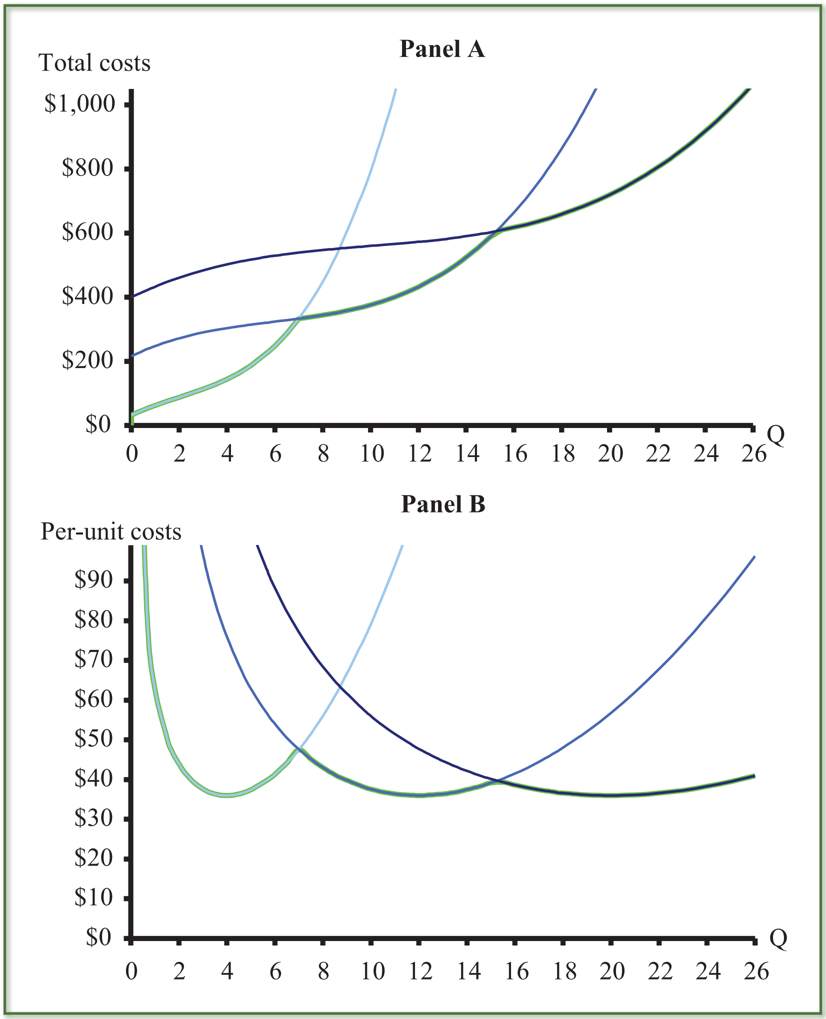
FIGURE 11.9 Long-Run Costs with Two Plant Sizes

you would choose the large plant if you expect to produce more than 8.5 units per time period. These choices produce the green LTC and LAC curves as the envelope of the blue STC and SATC curves.

The orientation of the two STC curves in Figure 11.9A accords with the expectation of fixed versus variable components of cost discussed in the previous paragraph. The small plant has smaller fixed cost and larger variable cost than the large plant. The relative size of the fixed component is seen in the vertical axis intercept for the two curves; as noted earlier, $FC_{small} = \$32 < \$400 = FC_{large}$. We argued that a smaller plant size would likely achieve a point of maximum labor productivity (point of inflection) at a smaller output level than a larger plant size. The point of inflection for the large plant size is at Q = 10; it turns out that the point of inflection for the small plant occurs at Q = 2, although the only thing that is clear from Figure 11.9 alone is that the quantity where this occurs is "small." Further, the variable cost curve is steeper for the small plant than for the large plant. These facts suggest that the small plant will be more labor intensive (or variable factor intensive) than the large plant, which will therefore be more capital intensive at producing the same level of output.

Suppose, now, that a third plant size is considered—one that is in between the small and large plants. The long-run curve with three available plant sizes is shown in Figure 11.10. Given these three plants, if output is expected to be less than about 7, the small plant would be chosen; if output is expected to be between 7 and 15, the medium plant would be chosen; and if output is expected to be larger than 15, the large plant would be chosen. The long-run total cost and long-run average cost curves are the envelope of short-run curves, just as with two available plant sizes. The only difference is that now there is an additional "scallop" to the curve, caused by the introduction of a new plant size.

FIGURE 11.10 Long-Run Costs with Three Plant Sizes



If we assume that we can create a variety of plant sizes, then we can "fill in" more sizes to create a smooth (not scalloped), continuous, long-run total cost and long-run average cost curve. This continuum of plant sizes is implicit in the production functions we examined in Chapters 9 and 10 (since K was continuous), and it is implicit in the smooth LREP discussed in Figure 10.10.[5] Figure 11.11 depicts how different plant sizes might fill in with intermediate-sized plants to those that existed in Figure 11.10. Now, five plant sizes are explicitly shown; each plant produces a different level of output at the same minimum short-run average total cost of production. The plants produce 4, 8, 12, 16, and 20 units of output at a minimum average total cost of $36 per unit. And more plants could be added in between each of these as well. The wide, light-blue LTC curve shown in Figure 11.11A is a straight line with a slope of $36. It is the envelope of an assumed continuum of plants that each produces different levels of output at minimum total cost. Similarly, the wide, light-blue LAC curve in Figure 11.11B is a horizontal line at a cost of $36/unit.

The envelope is a useful way to view long-run cost because each long-run choice returns us to a short-run situation. The long-run decision is the decision of which plant size to choose, and that is based on expected output. The envelope reminds us that this is actually a series of short runs. Different capital levels will be chosen, depending on what the firm expects to produce in the future. Once a capital size is chosen, the firm returns to a short run because the capital stock has been determined. A useful way to view this relation is to say:

The long run is the envelope of potential short runs.

The linearity of the LTC curve in Figure 11.11A is the result of a constant returns to scale (CRTS) production technology. In the present context, the easiest way to consider

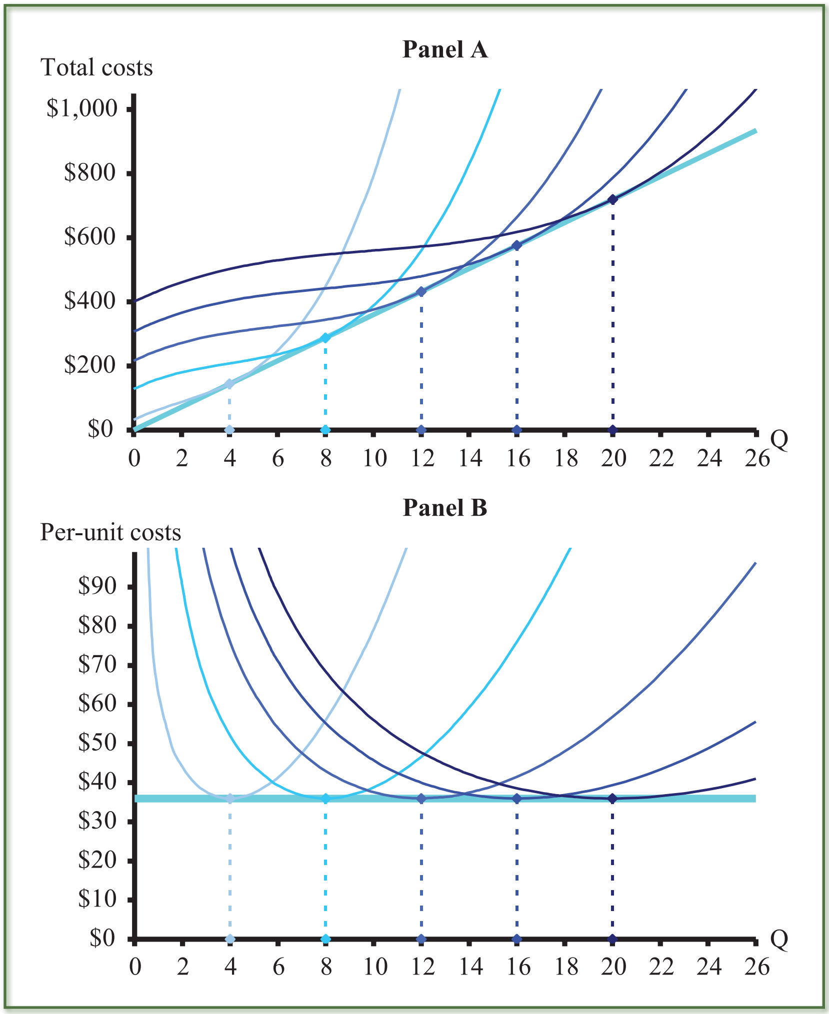


FIGURE 11.11 Long-Run Costs as the Envelope of Short-Run Costs with a Continuum of Plant Sizes: Constant Returns to Scale (CRTS)

returns to scale is to consider the cost of doubling output. With CRTS, doubling output doubles cost, but it only doubles cost in the long run because, in the short-run, capital cannot change. Indeed, in the short run, costs more than double.

We cannot examine returns to scale with a short-run cost function because the short-run cost function does not allow all factors to vary as required by the definition of returns to scale. We can, however, answer a different question in the short run:

Does a proportionate increase in output require a less-than-proportionate, proportionate, or more-than-proportionate increase in cost?

This question is answered according to whether the SATC is downward sloping, flat, or upward sloping at the output level under consideration. The proof of this assertion is straightforward. Consider the ratio of proportional change in cost to proportional change in output:

$$\%\Delta STC(Q)/\%\Delta Q = (\Delta STC(Q)/STC(Q))/(\Delta Q/Q). \quad \textbf{(11.3a)}$$

The left-hand side is less than one if costs increase less than proportionately with output changes, and it is greater than one if the reverse holds true. Regrouping the terms on the right-hand side of Equation 11.3a, we obtain:

$$\%\Delta STC(Q)/\%\Delta Q = [\Delta STC(Q)/\Delta Q]/[STC(Q)/Q]. \quad \textbf{(11.3b)}$$

The first bracketed term (the numerator) of the right-hand side of Equation 11.3b is SMC(Q), and the second bracketed term (the denominator) is SATC(Q). Therefore, Equation 11.3b simplifies to:

$$\%\Delta STC(Q)/\%\Delta Q = SMC(Q)/SATC(Q). \quad \textbf{(11.3c)}$$

We see from Equation 11.3c that costs increase less than proportionately with output change if SMC(Q) < SATC(Q) (so the change in cost to change in output ratio is less than one). This occurs for output levels below the minimum average total cost level of output. Similarly, costs increase more than proportionately with output changes when output is larger than the minimum average total cost level of output. This restates the relation between average and marginal discussed in Sections 9.2 and 11.1 (marginal pulls average up or down, depending on whether marginal is above or below average). A quick reflection on the meaning of average total cost verifies this result. Total cost increases less than proportionately if average total cost is declining (since total cost is average total cost times quantity; if there is a proportional increase in quantity but a decline in average total cost, then the product [of average total cost and quantity] is a less-than-proportional increase in total cost).

As discussed in Chapter 9, long-run production processes often exhibit increasing returns to scale for small levels of output and decreasing returns for large-scale production. If we assume a continuum of plant sizes (as we did for Figure 11.11), then we obtain smooth, long-run total cost and long-run average cost curves in this situation. Figure 11.12 shows a long-run total cost and long-run average cost curve consistent with this scenario. Also included are four short-run plant sizes; each short run is labeled in the graph according to the output level where there is tangency between the short-run and long-run curves. For example, K2 is the plant size that produces 2 units of output at minimum cost, K4 produces 4 units of output at minimum cost, etc.[6] The short-run curves holding capital fixed are denoted STC(Q; K_i) or SATC(Q; K_i), where "i" is the output level where tangency between short- and long-run curves occurs (capital after the semicolon means that capital is fixed). The economic interpretation of the tangency is that this plant size produces this level of output at minimum cost. The Excel file for Figure 11.12 includes larger versions of each diagram because fixed-cost differences for each of the short-run total cost curves are difficult to see in this instance.

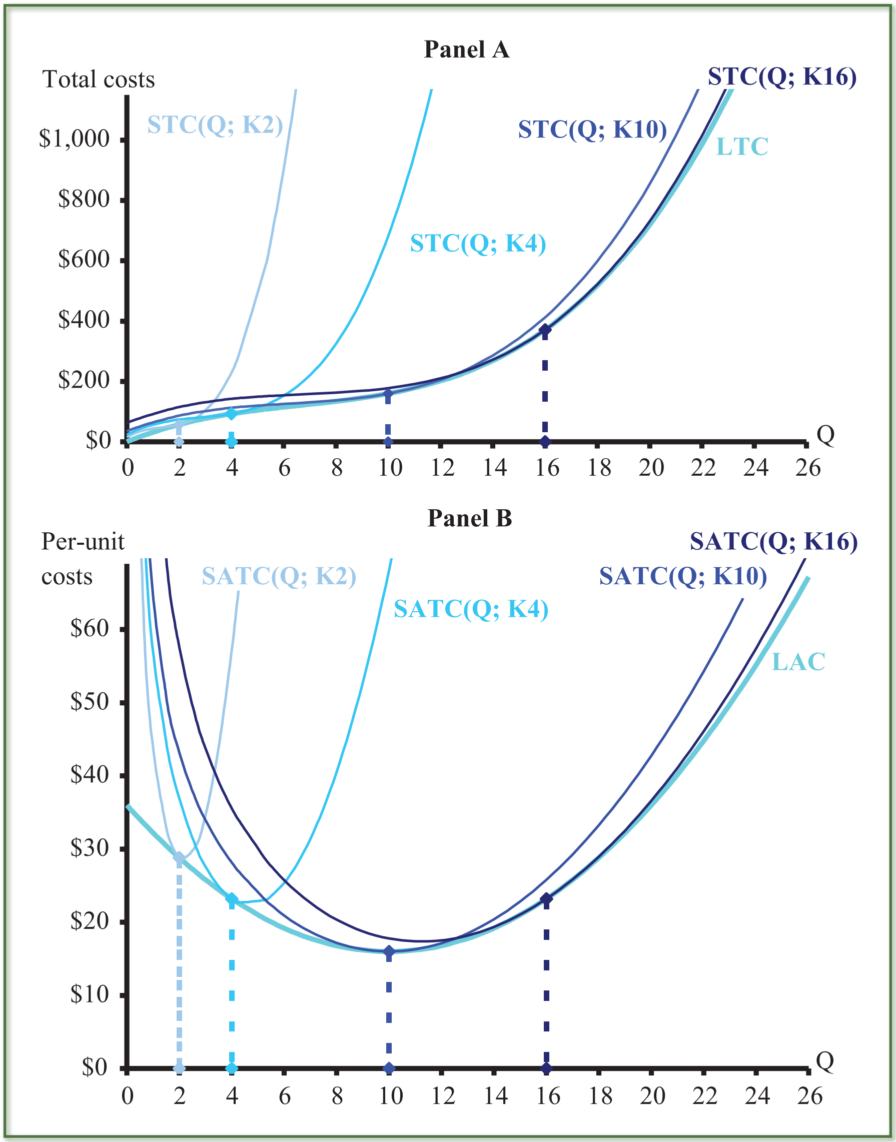


FIGURE 11.12 Long-Run Costs as the Envelope of Short-Run Costs with a Continuum of Plant Sizes: Economies and Diseconomies of Scale

The Geometry of Long-Run Marginal Cost (LMC)

The long-run marginal cost curve is the slope of the long-run total cost curve. Long-run marginal cost (LMC) can be derived from short-run cost functions, but *LMC is not the envelope of SMCs* (unlike LTC and LAC). As we will see, these two methods of deriving LMC are interrelated.

When production exhibits constant returns to scale, the long-run total cost curve is linear; therefore, long-run marginal cost is flat. Figure 11.11B can depict this situation if one change is made: LAC should be relabeled LMC. If LTC is linear, the slope of the chord and the slope of the tangent are the same. Both have a slope of \$36 in Figure 11.11A; therefore, LAC(Q) = LMC(Q) = \$36 in Figure 11.11B.

In contrast, when returns to scale vary, long-run total cost has the upward-sloping "S" shape shown in Figure 11.12A. LTC has a shape similar to SVC, so long-run marginal cost looks much like an SMC curve (just as LAC looks much like SAVC). Therefore, LMC and LAC are U-shaped, and just as with the short run, LMC = LAC at minimum LAC. This is shown in Figure 11.13. *Although the curves in both diagrams have the same shape as those in Figure 11.6A, they have entirely different conceptual meanings.* The

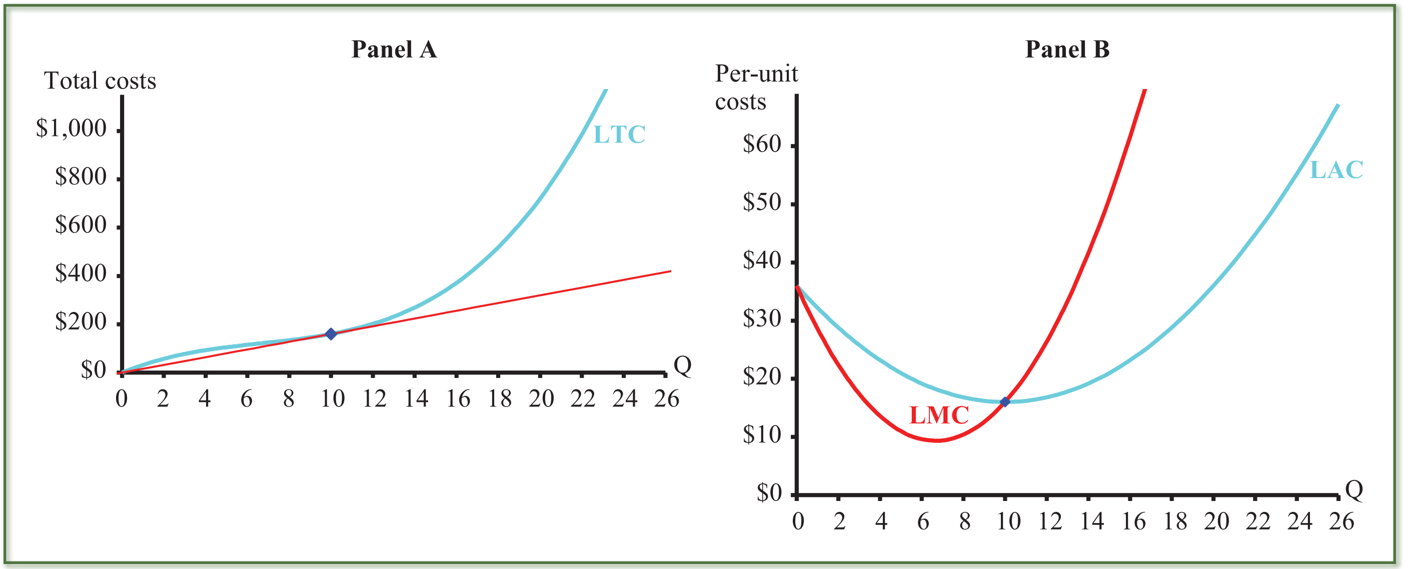


FIGURE 11.13 Long-Run Marginal Cost (LMC) as Slope of Long-Run Total Cost (LTC)

shape of SVC(Q) in the upper diagram in Figure 11.6A is due to varying labor productivity, given fixed capital. More specifically, the shape of SVC(Q) is determined by the cost of producing each level of output, using the short-run expansion path in Figure 10.10. The shape of LTC(Q) in Figure 11.13A is due to varying returns to scale (that occurs as all factors of production vary). Long-run average cost equals long-run marginal cost at minimum long-run average cost. Also like the short run, the slope of the minimum sloping chord to the LTC curve equals the slope of the tangent to that curve at that point. This occurs at the blue diamond tangency point (Q, \$) = (10, \$160) in Figure 11.13A; therefore, LAC(10) = LMC(10) = \$16 in Figure 11.13B.

Suppose you wish to produce Q* units of output at minimum long-run cost. In this instance, you would be on the LTC curve at the point Q = Q*. Since LTC is the envelope of short-run curves, there must be a short-run plant size, K*, that produces this output level; LTC(Q*) = STC(Q*; K*). Dividing by Q*, we obtain:

$$LAC(Q^*) = SATC(Q^*; K^*). \qquad \textbf{(11.4a)}$$

This is seen geometrically by the coincidence of LAC and different SATCs at Q* = 2, 4, 10, and 16, given plant sizes K2, K4, K10, and K16 in Figure 11.12B. We actually know more, since the LTC curve is tangent to each of the STC curves at these output levels in Figure 11.12A. Tangency at this point means that the slope of each curve is the same, and slope is marginal cost. Therefore, we may formally state this as:

$$LMC(Q^*) = SMC(Q^*; K^*). \qquad \textbf{(11.4b)}$$

Long-run marginal cost is seen as the short-run marginal cost associated with the capital stock that produces that output level at minimum cost.

Long-run average cost is equal to the associated short-run average total cost (Equation 11.4a), and long-run marginal cost is equal to the associated short-run marginal cost (Equation 11.4b). Short- and long-run costs are seen to be pair-wise equal; average equals average and marginal equals marginal, according to Equations 11.4a and 11.4b.

The pair-wise equality between short-run and long-run marginal cost from Equation 11.4b is seen in Figure 11.14 for two output levels, Q* = 4 and Q* = 16 (the Q* = 16 versions of Equations 11.4a and 11.4b are explicitly noted on Figure 11.14).

The tangency between LTC and STC at these output levels in Figure 11.14A represents the coincidence of LAC and SATC at these output levels in Figure 11.14B. (This tangency in Figure 11.14A and coincidence in Figure 11.14B are geometric representations of Equation 11.4a.) The point (4, SMC(4; K4)) = (4, \$13.60) on the gold SMC(Q; K4) curve equals

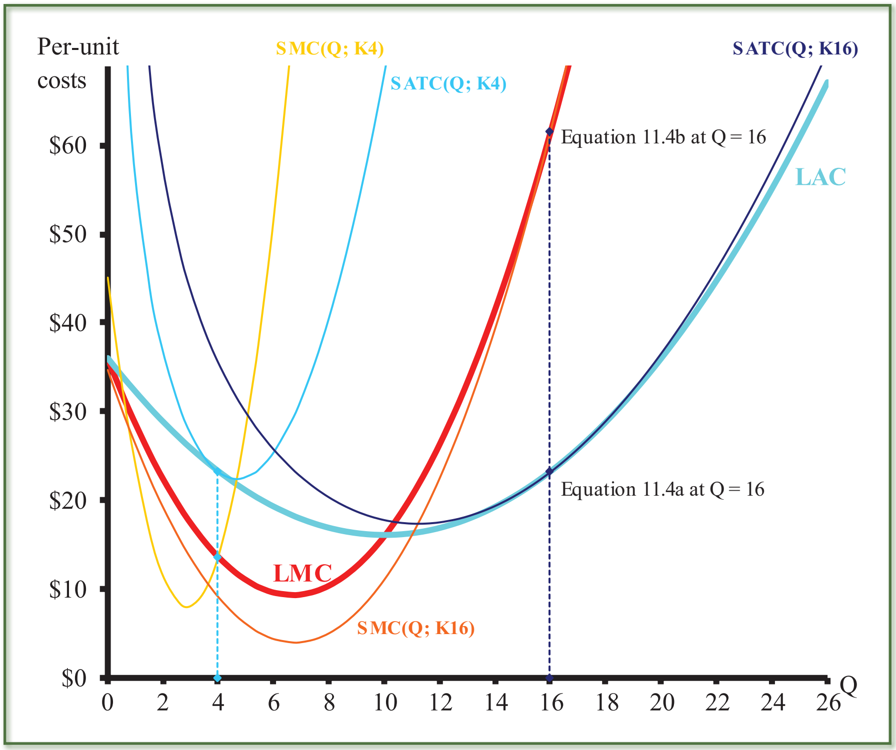


FIGURE 11.14 Long-Run Marginal Cost from Short-Run Per-Unit Costs

LMC is *not* the envelope of SMCs.

LMC(4), according to Equation 11.4b, and the point (16, SMC(16; K16)) = (16, $61.60) on the orange SMC(Q; K16) curve equals LMC(16), according to Equation 11.4b.[7] This shows how LMC can be constructed from short-run per-unit cost curves.

Students commonly believe that a given plant size must be run at minimum average cost to be producing at a point on the long-run cost function. The following example shows that this belief is false. Consider the K16 plant size that produces 16 units of output at minimum average cost of $23.20 per unit in Figure 11.14. Marginal cost in this instance is substantially higher ($61.60 as stated previously), and therefore producing a smaller level of output reduces average cost, given this short-run production process. Minimum SATC(Q; K16) occurs at Q =11, and SATC(11; K16) = SMC(11; K16) = $17.38. This plant produces 16 units of output at minimum cost, but it produces 11 units of output even more cheaply (on the basis of ATC comparisons). Of course, there is a plant size, K11, that would produce 11 units of output even more cheaply than the K16 plant; this plant size would be between K10 and K16 in size (assuming a continuum of plant sizes). Similarly, the K4 plant size produces output at lower average cost if Q = 4.67 units are produced than if 4 units are produced.

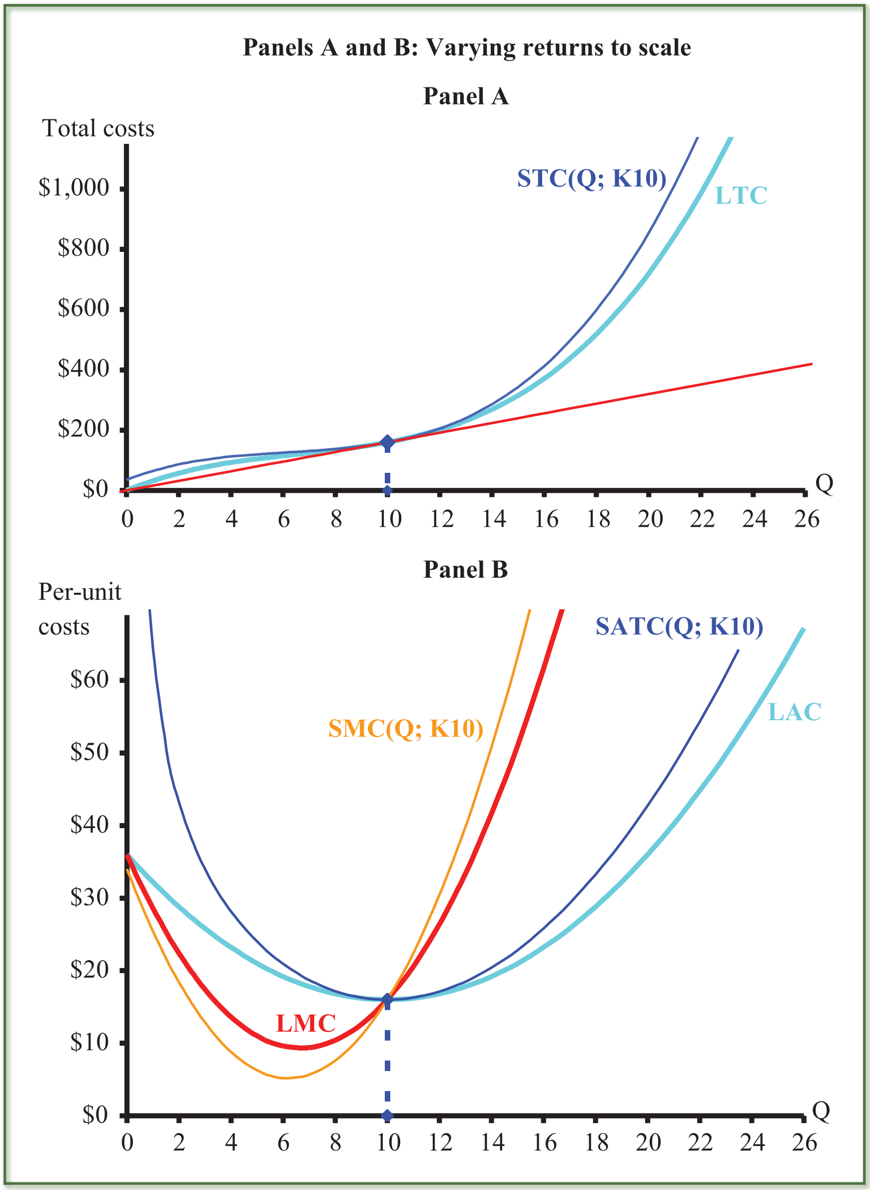
The belief that a given plant size must be run at minimum average cost to be producing at a point on the long-run cost function is unfounded. The plants that produce 4 and 16 units in the most cost-effective manner are, in a more global sense, not producing at least cost. The same is true for other plant sizes as well, and it will continue to be true if the production point under consideration is *not* a point of minimum long-run average cost.

Equations 11.4a and 11.4b require pair-wise equalities at minimum-cost production of a given level of output. *But these pair-wise equalities are also equal to each other when Q* is an output level that produces output at minimum long-run average cost.* This can be stated algebraically as:

$$\text{SATC}(Q^*; K^*) = \text{minimum LAC}(Q^*) = \text{LMC}(Q^*) = \text{SMC}(Q^*; K^*). \quad \textbf{(11.5)}$$

The first equality in Equation 11.5 is required by Equation 11.4a, and the third is required by Equation 11.4b. The second is the relation that exists between average and marginal, discussed in Chapters 9 and 11: Average equals marginal only at maximums or minimums of average. In Chapter 9, marginal product passes through the maximum of average

FIGURE 11.15
Minimum LAC: The Point Where LAC = LMC = SMC = SATC



product (as explicitly shown in Figure 9.3); in this chapter, marginal cost passes through the minimum of average cost (as shown in Figures 11.6 and 11.13). The coincidence of these four cost curves shown in Figure 11.15B is based on the varying returns to scale, U-shaped, long-run average cost curve examined in Figures 11.12–11.14. With the varying returns to scale (the cost structure shown in Figures 11.15A and 11.15B), Equation 11.5 holds at a single, long-run, average cost-minimizing output level—in this event, Q* = 10.

If the production function exhibited constant returns to scale (as in Figures 11.9–11.11), then *each* point on the LAC(Q) curve would be associated with a plant size for which Equation 11.5 would hold. The reason is straightforward: If LAC is flat, then LAC(Q) = LMC(Q) for every output level. The center equality in Equation 11.5 holds for all output levels, while the first and third equalities hold at each output level for the plant size that produces that output at minimum cost, according to Equations 11.4a and 11.4b. This is shown in Figures 11.15C and 11.15D. These figures show five output levels (Q* = 4, 8, 12, 16, and 20) where Equation 11.5 holds, although the same condition holds at other output levels as well (if the capital stock is appropriately adjusted).

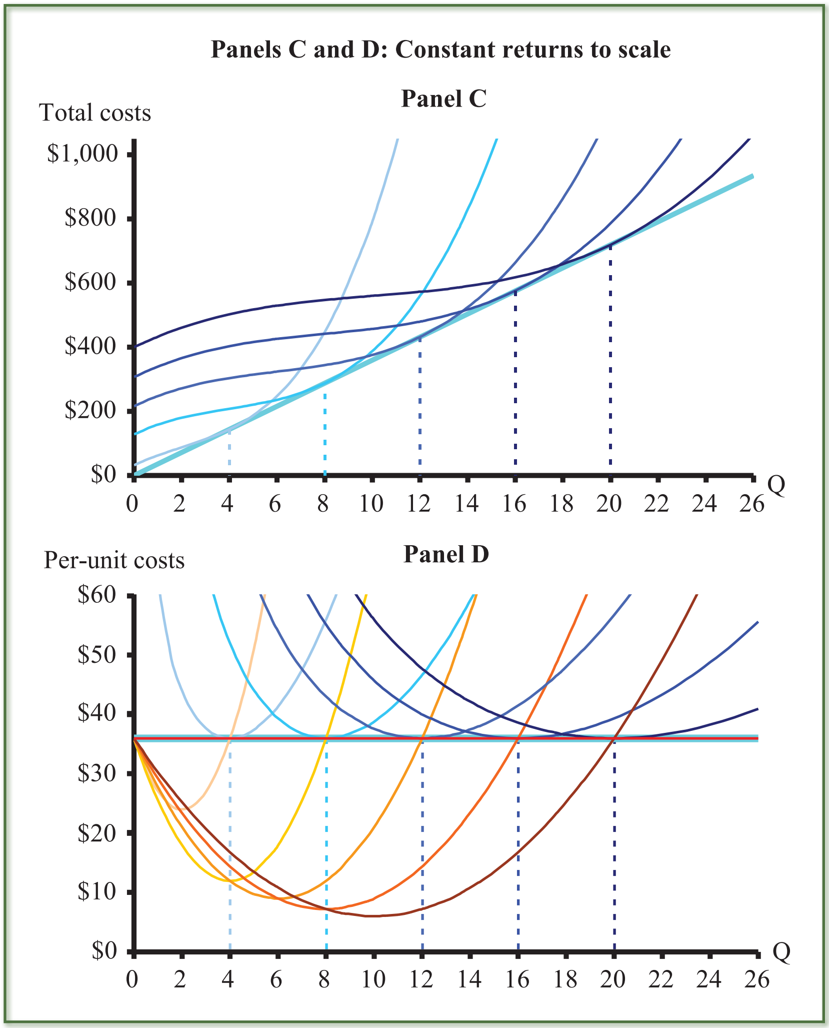


FIGURE 11.15
continued

Key: STC and SATC curves are different gradations of blue; SMC curves are different gradations of gold.

Long-Run Cost with Discrete Plant Sizes *OPTIONAL SECTION*

When production can only be accomplished with discrete plant sizes, the LTC and LAC curves are scalloped, as described in Figures 11.9 and 11.10. Most basically, long-run marginal cost is the slope of long-run total cost, as described in Figure 11.13, and therefore, the long-run marginal cost curve would have jump discontinuities at the output levels where the lowest cost method of production switches from one size plant to the next. For production levels just below this critical level of output, the smaller plant size is utilized, and LMC equals the SMC for the smaller plant. For production levels just above this output, the larger plant size is utilized, and LMC equals the SMC for the larger plant. Both of these marginal costs are different from each other because the two STC curves have different slopes at the point of intersection. This is what causes the LTC curve to be scalloped in Figures 11.9 and 11.10. At other output levels, the LTC, LMC, and LAC equal the short-run curves that minimize production cost for that output level.

The cost curves with discrete plant sizes depend, of course, on the plant sizes that are available. Figure 11.16 shows LAC(Q) and LMC(Q), given four plant sizes—K2, K4, K10, and K16—for the variable returns process depicted in Figures 11.12–11.15A and 11.15B. Figure 11.17 shows the same per-unit cost curves for the five discrete plant sizes used to depict Figures 11.9–11.11 and Figures 11.15C and 11.15D. Both figures exhibit jump discontinuities at crossover output levels as expected. The Excel files for Figures 11.16 and 11.17 allow you to view the LTC curves for both production processes, but these LTC curves are scalloped as expected, based on Figures 11.9 and 11.10. The scalloping of LTC causes scalloping of LAC and jumps in LMC for the reason discussed earlier.

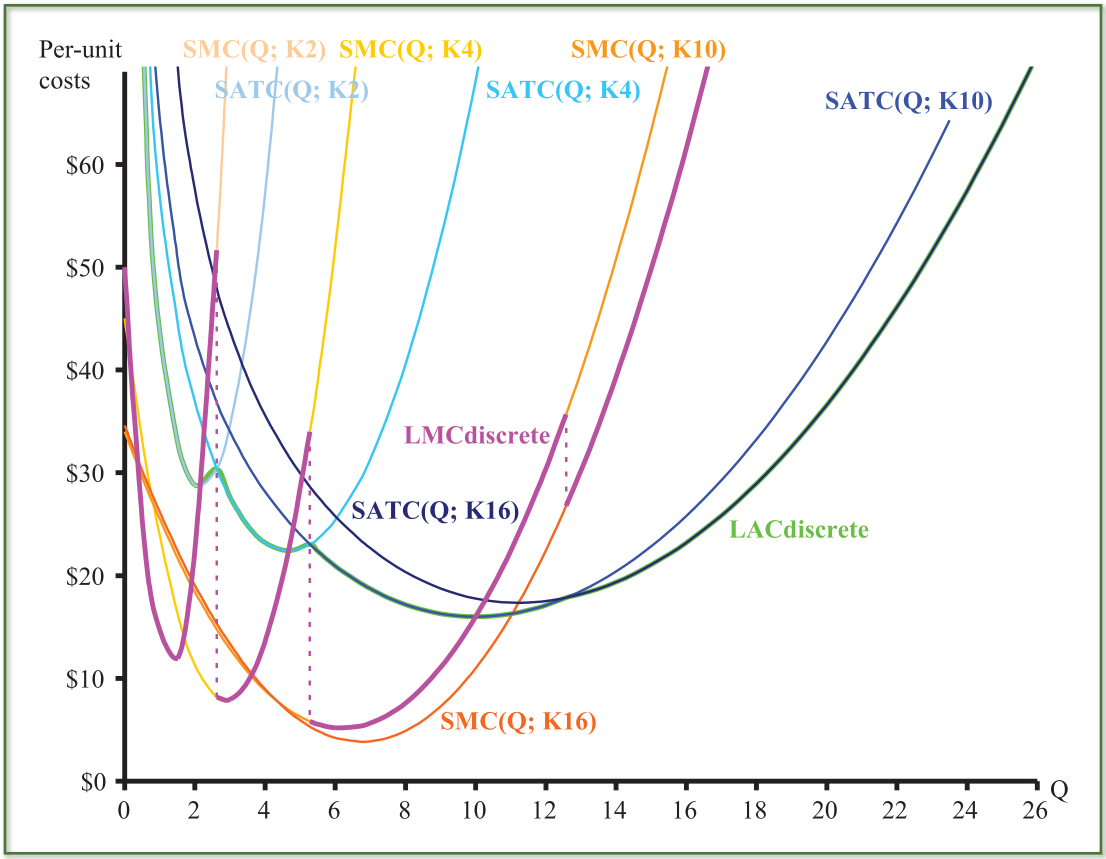


FIGURE 11.16 Long-Run Marginal Cost with Discrete Plant Sizes and Varying Returns to Scale

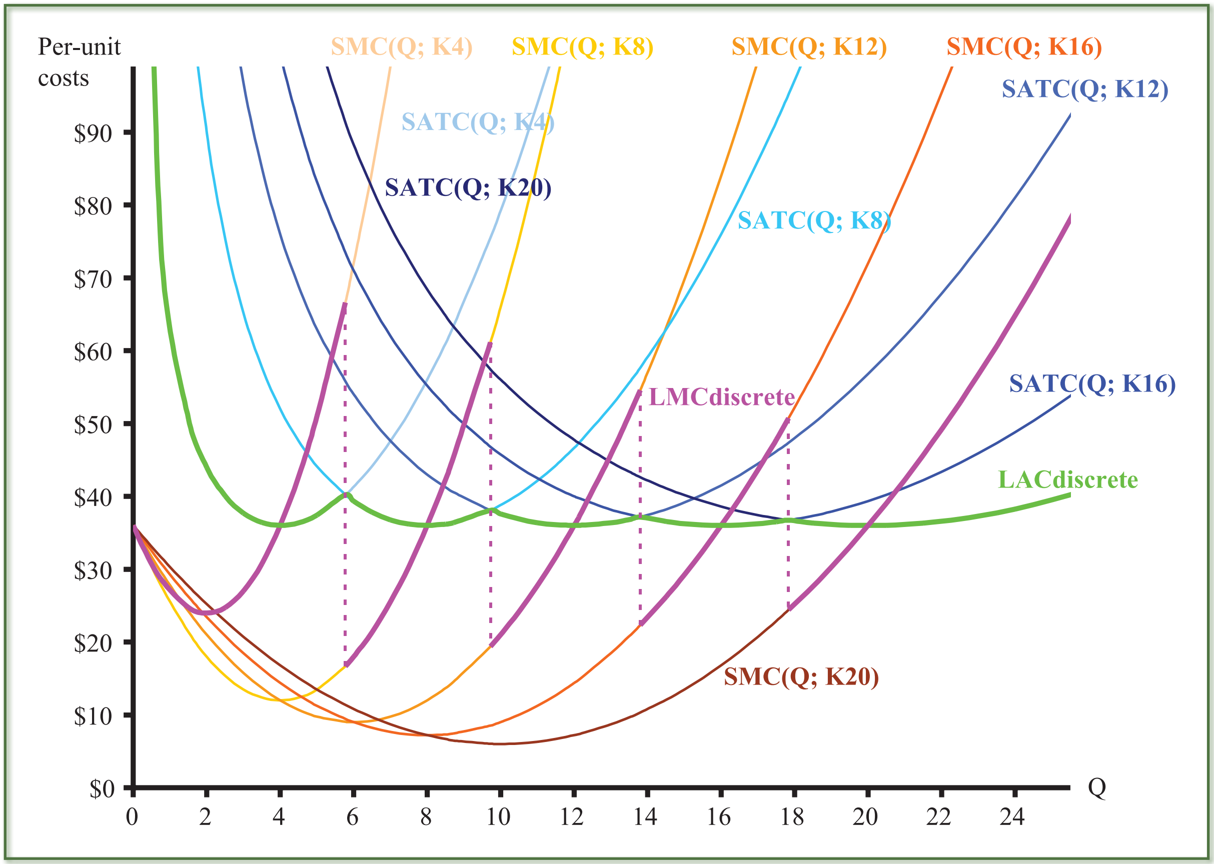


FIGURE 11.17 Long-Run Marginal Cost with Discrete Plant Sizes and Constant Returns to Scale

11.4 The Algebra of Cubic Cost Functions

The cost curves depicted thus far in this chapter are based on the cubic functional form. Cost functions need not be cubic, but this form offers the simplest algebraic means of describing a total or variable cost curve that is first convex downward and then convex upward. As described in Section 11.2, a typical short-run cost curve exhibits this changing convexity due to changing marginal productivity of the variable factors of production. Initial increasing marginal productivity is eventually replaced by declining marginal productivity, due to the law of diminishing marginal product. In Section 11.3, we argued that the long-run total cost curve may also have this shape, due to varying returns to scale. This would occur if initial increasing returns to scale are eventually replaced by decreasing returns to large-scale production.

Both short-run and long-run cost curves may be depicted algebraically, using the cubic functional form. We focus on the seven short-run curves delineated in Section 11.2 because, as argued earlier, in the discussion of Figure 11.12, LTC, LAC, and LMC curves based on varying returns to scale are *geometrically* identical to SVC, SAVC, and SMC. The general form of the cubic total cost function is:

$TC(Q) = a + b \cdot Q - c \cdot Q^2 + d \cdot Q^3$, where a, b, c, and d are positive numbers. **(11.6a)**

Before continuing, we should note a couple of points. First, there is a minus sign in front of the quadratic term c and plus signs in front of the linear and cubic terms. This is what produces U-shaped average and marginal cost curves. Second, we explicitly stated at the start of the paragraph that we would focus on short-run costs. As a result, the "S" is implicit for each of the cost functions in Equations 11.6a–11.6g. Third, this is the short run because parameter *a* represents fixed cost.

$FC(Q) = a$, given the total cost function in Equation 11.6a. **(11.6b)**

Each of the other five short-run cost functions (VC, ATC, AVC, AFC, MC) is obtained by algebraic manipulation of Equation 11.6a.

Since *a* is fixed cost, and total cost is variable cost plus fixed cost (Equation 11.1a), the rest of Equation 11.6a is variable cost.

$VC(Q) = b \cdot Q - c \cdot Q^2 + d \cdot Q^3$, given the total cost function in Equation 11.6a. **(11.6c)**

The short-run average cost functions are obtained by dividing each function by Q.

$ATC(Q) = a/Q + b - c \cdot Q + d \cdot Q^2$. **(11.6d)**

$AFC(Q) = a/Q$. **(11.6e)**

$AVC(Q) = b - c \cdot Q + d \cdot Q^2$. **(11.6f)**

As expected, TC(Q) = FC(Q) + VC(Q) and ATC(Q) = AFC(Q) + AVC(Q).

Marginal cost is the slope of the total cost and variable cost curves. Given the total cost curve in Equation 11.6a or the variable cost curve in Equation 11.6c, marginal cost is given by:

$MC(Q) = b - 2 \cdot c \cdot Q + 3 \cdot d \cdot Q^2$. **(11.6g)**

Equation 11.6g describes the slope of total cost and variable cost, and can be found by taking the derivative of either TC(Q) or VC(Q) in Equations 11.6a or 11.6c. If you have not seen calculus before, then this is simply a fact that can be used whenever you have a cubic cost function.

Claim: *You can obtain marginal cost from a cubic cost function by applying Rules 1 and 2 to the total cost function:*

Rule 1—Drop the fixed cost component *a*; this only shifts total cost up or down, but does not change slope at a given output level.

Rule 2—The linear, quadratic, and cubic terms (b, c, and d) become the constant, linear, and quadratic coefficients, once they are multiplied by 1, 2, and 3.

This simple "cookbook" method will always provide you with marginal cost.[8]

AVC and MC in Equations 11.6f and 11.6g have been placed next to each other so that it is easy to see the similarity between these functions. MC looks like AVC, except that the last two terms are multiplied by 2 and 3 (using Rule 2 in the previous "cookbook" method).

Each of the U-shaped per-unit cost curves attains a minimum value. Two of the three minimum values are easy to obtain. Minimum marginal cost occurs at:[9]

$$Q_{minMC} = c/(3 \cdot d). \quad \textbf{(11.7a)}$$

Minimum AVC may be obtained algebraically using the fact that MC = minimum AVC (as shown in Figure 11.6A). Using Equations 11.6f and 11.6g we obtain upon simplification:

$$Q_{minAVC} = c/(2 \cdot d). \quad \textbf{(11.7b)}$$

Comparing Equations 11.7a and 11.7b, we see that when costs are cubic, minimum MC occurs at two-thirds the output of minimum AVC. This is a helpful fact for graphing these two per-unit cost functions.

Unfortunately, if you follow the same strategy to find minimum average total cost (ATC), you obtain a cubic equation. The general solution of cubic equations is beyond the scope of this text. However, we can find a numerical solution, once we know *a*, b, c, and d, by comparing ATC(Q) and MC(Q) for different values of Q. (When ATC(Q) > MC(Q) at the given Q, increase Q; when ATC(Q) < MC(Q) at a given Q, decrease Q.) This will quickly produce as close of a numerical solution as necessary.[10] (And of course, problems produced for exams will often have clean solutions that need little numerical approximation. Once you know *a*, b, c, and d, set ATC(Q) = MC(Q), and look to see if an "easy" solution is visible—it often is, and it gets easier to see the more often you look for it.)

It is worthwhile to apply this to a specific cubic cost function. Consider, for instance, the cost function used to produce each of the figures in Section 11.2. We derived each of the minimum values graphically; we now confirm these answers algebraically. The cost function used is:

$$TC(Q) = 400 + 36 \cdot Q - 3 \cdot Q^2 + 0.1 \cdot Q^3. \quad \textbf{(11.8a)}$$

Focus on the U-shaped per-unit cost curves.

$$ATC(Q) = 400/Q + 36 - 3 \cdot Q + 0.1 \cdot Q^2. \quad \textbf{(11.8b)}$$

$$AVC(Q) = 36 - 3 \cdot Q + 0.1 \cdot Q^2. \quad \textbf{(11.8c)}$$

$$MC(Q) = 36 - 6 \cdot Q + 0.3 \cdot Q^2. \quad \textbf{(11.8d)}$$

The similarity between AVC and MC just described applies with numbers in place of parameters (as you would expect). Applying Equations 11.7a and 11.7b to this cost function, we see that minimum marginal cost and average variable cost occur at:

$$Q_{minMC} = c/(3 \cdot d) = 3/(3 \cdot 0.1) = 10. \quad \textbf{(11.9a)}$$

$$Q_{minAVC} = c/(2 \cdot d) = 3/(2 \cdot 0.1) = 15. \quad \textbf{(11.9b)}$$

Note that, as expected, minimum MC occurs at two-thirds the output level of minimum AVC.

As stated earlier, the general solution for minimum ATC(Q) requires the solution of a cubic equation. The strategy suggested in the last paragraph provides the following (by equating ATC to MC):

$$400/Q + 36 - 3 \cdot Q + 0.1 \cdot Q^2 = 36 - 6 \cdot Q + 0.3 \cdot Q^2. \quad \textbf{(11.9c)}$$

Combining common terms and putting all terms on one side of the equality yields:

$$0.2 \cdot Q^2 - 3 \cdot Q - 400/Q = 0. \quad \textbf{(11.9d)}$$

Multiplying by 5Q, we obtain:

$$Q^3 - 15 \cdot Q^2 - 2000 = 0. \quad \textbf{(11.9e)}$$

TABLE 11.1 Using Excel to Find Minimum ATC, Given Cubic Cost

Panel A: ATC versus MC for an initial value of output

	A	B	C	D
1	Q	ATC(Q) (Equation 11.8b)	MC(Q) (Equation 11.8d)	ATC-MC
2	15	40.16666667	13.5	26.6666667
3	Row 2 Equations	=400/A2+36-3*A2+0.1*A2^2	=36-6*A2+0.3*A2^2	=B2-C2

Panel B: Minimum ATC—a numerical solution achieved by Goal Seek

	A	B	C	D
1	Q	ATC(Q) (Equation 11.8b)	MC(Q) (Equation 11.8d)	ATC-MC
2	19.99999981	36	35.99999886	1.14E-06
3	Row 2 Equations	=400/A2+36-3*A2+0.1*A2^2	=36-6*A2+0.3*A2^2	=B2-C2

Panel C: Minimum ATC—an exact solution obtained by typing 20 in cell A2

	A	B	C	D
1	Q	ATC(Q) (Equation 11.8b)	MC(Q) (Equation 11.8d)	ATC-MC
2	20	36	36	0
3	Row 2 Equations	=400/A2+36-3*A2+0.1*A2^2	=36-6*A2+0.3*A2^2	=B2-C2

This equation (or its precursors, Equations 11.9c and11.9d) can be checked for values of Q to determine the minimum ATC quantity level. Any Q chosen must, of course, be larger than that found in Equation 11.9b, since $Q_{minAVC} < Q_{minATC}$. Substituting Q values larger than Q = 15 will quickly produce Q = 20 as a solution. This search can be done manually or by programming the equations into Excel and using the **Goal Seek** function (described in Section 7.5) to obtain the answer. If checking manually, it is worthwhile to work from Equation 11.9c, rather than the cubic equation in Equation 11.9e, since the information provided by the relative sizes of ATC(Q) and MC(Q) tells us whether to increase or decrease Q in the next iteration of our search. The directional change rule is based on the relation that governs the relative size of marginal and average cost discussed earlier: *When ATC(Q) > MC(Q) at a given Q, increase Q; when ATC(Q) < MC(Q) at a given Q, decrease Q.* Not surprisingly, the answers obtained in Equation 11.9 mirror those obtained graphically in Section 11.2.

Table 11.1 shows the Excel version of this search. Table 11.1A sets out the equations for ATC and MC from Equations11.8b and 11.8d in cells B2 and C2, based on the quantity in cell A2 (the equations in these cells are shown beneath in cells B3 and C3). Table 11.1B shows the results of a **Goal Seek** on cell D2 by changing cell A2. (Click on cell D2, click **Data, What If Analysis, Goal Seek** in Excel 2007 (or **Tools, Goal Seek** in Excel 2003). Put 0 (zero) in the **To value** area and A2 in the **By changing cell** area, click **OK, OK**.) The result is Q=19.99999981, a number that is *very* close to 20. You may wonder why 20 was not obtained as the solution in this instance. **Goal Seek** is a numerical search routine that stops when an answer "close enough" to the goal is achieved. As you can see in cell D2, ATC and MC are not equal, but they are within 0.00000116 of each other! Table 11.1C depicts the solution obtained by putting the value Q = 20 in cell A2. In this instance, the answer is exact.

Interpreting and Restricting Cubic Cost Parameters

The cubic cost curves described in Equations 11.6a–11.6g change as *a*, b, c, and d vary. Two of these parameters, *a* and b, have easily seen intuitive meanings: *a* is fixed cost as noted earlier, and b is the initial level of marginal cost and average variable cost (according to Equations 11.6b, 11.6f, and 11.6g). If either of these parameters changes, you should be able to work through what happens to the various cost curves. The other parameters are less intuitive but have readily understood interpretations. Larger values of c will make the s shape of the cost curve more pronounced over the middle range of outputs, and larger values of d will make the cost curve steeper for large values of output. Changing the value of these parameters may lead to cubic functions that no longer represent a total cost curve because the shape is no longer a valid representation of cost as a function of output. A further restriction must be placed on the parameters b, c, and d to assure that the resulting function represents a possible cost curve.

The initial restriction on these parameters is that each must be positive. A restriction on the shape of the total cost curve is that it must be upward sloping (because it must be more expensive to produce more output than less output); put another way, marginal cost must be positive. Marginal cost in Equation 11.6g is greater than zero when the following holds:

$$c^2 < 3 \cdot b \cdot d. \qquad \textbf{(11.10)}$$

This restriction may appear a bit strange but it has an easy explanation. Marginal cost is a quadratic function of quantity, Q. Because we do not want MC(Q) = 0, the quadratic equation should have no solutions (you may recall from algebra that a quadratic equation can have zero, one, or two solutions).[11] A quick check of the total cost function described in Equation 11.8a confirms this condition ($3^2 = 9 < 3 \cdot 36 \cdot 0.1 = 10.8$).

When you create your own cubic cost functions, you will quickly know if you violate the inequality in Equation 11.10: Your total and variable cost curves will have a downward-sloping segment (or the slope will be zero at a point), and marginal cost will have output levels where it is less than zero (or zero). It would be worthwhile to spend some time adjusting parameters *a*–d to see what happens to the various cost curves, using the Excel file for the Chapter 11 Appendix.

11.5 Cobb-Douglas Cost Functions *OPTIONAL SECTION*

Cost functions based on the Cobb-Douglas production function $Q(L, K) = a \cdot L^b \cdot K^c$ have already been derived in Chapter 10. The long-run total cost function is derived as Equation 10.13a, and the short-run total cost function is Equation 10.16a, but they are reproduced here as Equations 11.11a and 11.12a to provide a more unified exposition. Each of the other cost curves discussed in Chapter 11 can be obtained from these two functions. Much can be gleaned from this section by focusing on the figures, rather than worrying about the algebra. In many cases, the second version of equations in this section shows the functional relation without the algebraic detail. Even without working through the algebra, it is worth opening the Excel file that produced all of the figures in this section and manipulating *a*, b, c, w, and r to see what happens to each of the cost curves as a result.

The long-run total cost function is:

$$LTC(Q) = Q^{(1/(b+c))} \cdot [a^{(-1/(b+c))} \cdot w^{(b/(b+c))} \cdot r^{(c/(b+c))} \cdot ((b/c)^{(c/(b+c)} + (c/b)^{(b/(b+c))})]. \qquad \textbf{(11.11a)}$$

$$LTC(Q) = \mathbf{Q^{(1/(b+c))}} \cdot H(w, r, a, b, c). \qquad \textbf{(11.11b)}$$

H is the shorthand term for the function in the brackets [ ] in Equation 11.11a. The "a" version is the actual cost equation; the "b" version shows the functional form implicit in the equation (LTC is a power function of quantity, Q). From here, the average cost function is easily obtained by dividing by Q:

$$LAC(Q) = \mathbf{Q^{((1/(b+c))-1)}} \cdot H(w, r, a, b, c). \qquad \textbf{(11.11c)}$$

The long-run marginal cost function associated with this cost curve is:[12]

$$LMC(Q) = \mathbf{Q^{((1/(b+c))-1)}} \cdot H(w, r, a, b, c)/(b + c). \qquad \textbf{(11.11d)}$$

The short-run cost function associated with having K_0 units of capital from Equation 10.16 is (in each case, we provide two versions of each equation to highlight the functional relation that connects quantity to cost):

$STC(Q; K_0) = r \cdot K_0 + Q^{(1/b)} \cdot [w \cdot (K_0^{-c}/a)^{(1/b)}]$.

$STC(Q; K_0) = F(r, K_0) + \mathbf{Q^{(1/b)}} \cdot G(w, a, b, c, K_0)$. **(11.12a)**

Short-run total cost is easily decomposed into fixed and variable components:

$SFC(Q; K_0) = r \cdot K_0$.

$SFC(Q; K_0) = F(r, K_0)$. **(11.12b)**

$SVC(Q; K_0) = Q^{(1/b)} \cdot [w \cdot (K_0^{-c}/a)^{(1/b)}]$.

$SVC(Q; K_0) = \mathbf{Q^{(1/b)}} \cdot G(w, a, b, c, K_0)$. **(11.12c)**

From here, each of the average cost functions (SATC, SAVC, and SAFC) is obtained by dividing each of the respective total functions by Q:

$SATC(Q; K_0) = r \cdot K_0/Q + Q^{((1/b)-1)} \cdot [w \cdot (K_0^{-c}/a)^{(1/b)}]$.

$SATC(Q; K_0) = SAFC + SAVC$. **(11.12d)**

$SAFC(Q; K_0) = r \cdot K_0/Q$.

$SAFC(Q; K_0) = F(r, K_0)/Q$. **(11.12e)**

$SAVC(Q; K_0) = Q^{((1/b)-1)} \cdot [w \cdot (K_0^{-c}/a)^{(1/b)}]$.

$SAVC(Q; K_0) = \mathbf{Q^{((1/b)-1)}} \cdot G(w, a, b, c, K_0)$. **(11.12f)**

Finally, short-run marginal cost is:

$SMC(Q; K_0) = (1/b) \cdot Q^{((1/b)-1)} \cdot [w \cdot (K_0^{-c}/a)^{(1/b)}]$.

$SMC(Q; K_0) = \mathbf{Q^{((1/b)-1)}} \cdot G(w, a, b, c, K_0)/b$. **(11.12g)**

The total cost functions were further examined, based on specific values of the parameters used to produce Figure 10.10; $a = 1$, $b = c = 0.5$, and $w = r = 1$. These values of a, b, c, w, and r produce the quadratic short-run total cost function and the linear long-run total cost function described in Equations 10.17a and 10.17b, reproduced here as Equations 11.13a and 11.13b.

LR cost: $LTC(Q) = 2 \cdot Q$ from Equation 11.11a. **(11.13a)**

SR cost: $STC(Q; K_0) = K_0 + Q^2/K_0$ from Equation 11.12a. **(11.13b)**

These cost functions are depicted in Figure 11.18A (the parameter values are listed to the left of Figure 11.18B). The long-run cost curves are shown together with short-run cost curves associated with four different plant sizes (K_i = 0.5, 1, 2, and 3). Due to the complexity of Figure 11.18B, only LAC, LMC, SATC, and SMC are shown. The result proven at Equation 10.18 based on these cost functions is that $STC(Q; K_0) \geq LTC(Q)$, and they are only equal when $Q = K_0$. This result is seen both in Figure 11.18A as the tangency of each of the short-run total cost curves with the long-run total cost curve, and in Figure 11.18B by the convergence of SMC, SATC, and LAC = LMC at Q = 0.5, 1, 2, and 3 (according to Equation 11.5). Per-unit costs are minimized at a cost of 2 per unit according to Equation 11.13a and by the height of LAC in Figure 11.18B.

The general total and per-unit Cobb-Douglas cost equations in Equations 11.11 and 11.12 are complex, but beneath the complexity are some readily apparent patterns. To understand these patterns, it is necessary to distinguish between two aspects of a curve: the basic *shape* of a curve and its *placement*. By *shape*, we mean: What is the functional form inherent in the function? How does cost relate to quantity? Is the curve convex upward, convex downward, or does it change convexity? By *placement*, we mean the graph's location horizontally and vertically on the graph. The vertical placement is the height of cost; the horizontal placement is how stretched out is the curve. Think of a curve drawn on a clear, elastic surface above a fixed piece of graph paper. If you fix the horizontal axis and pull upward on the top of the piece of elastic, the shape of the curve will not change, but its vertical placement relative to the fixed graph paper will change.

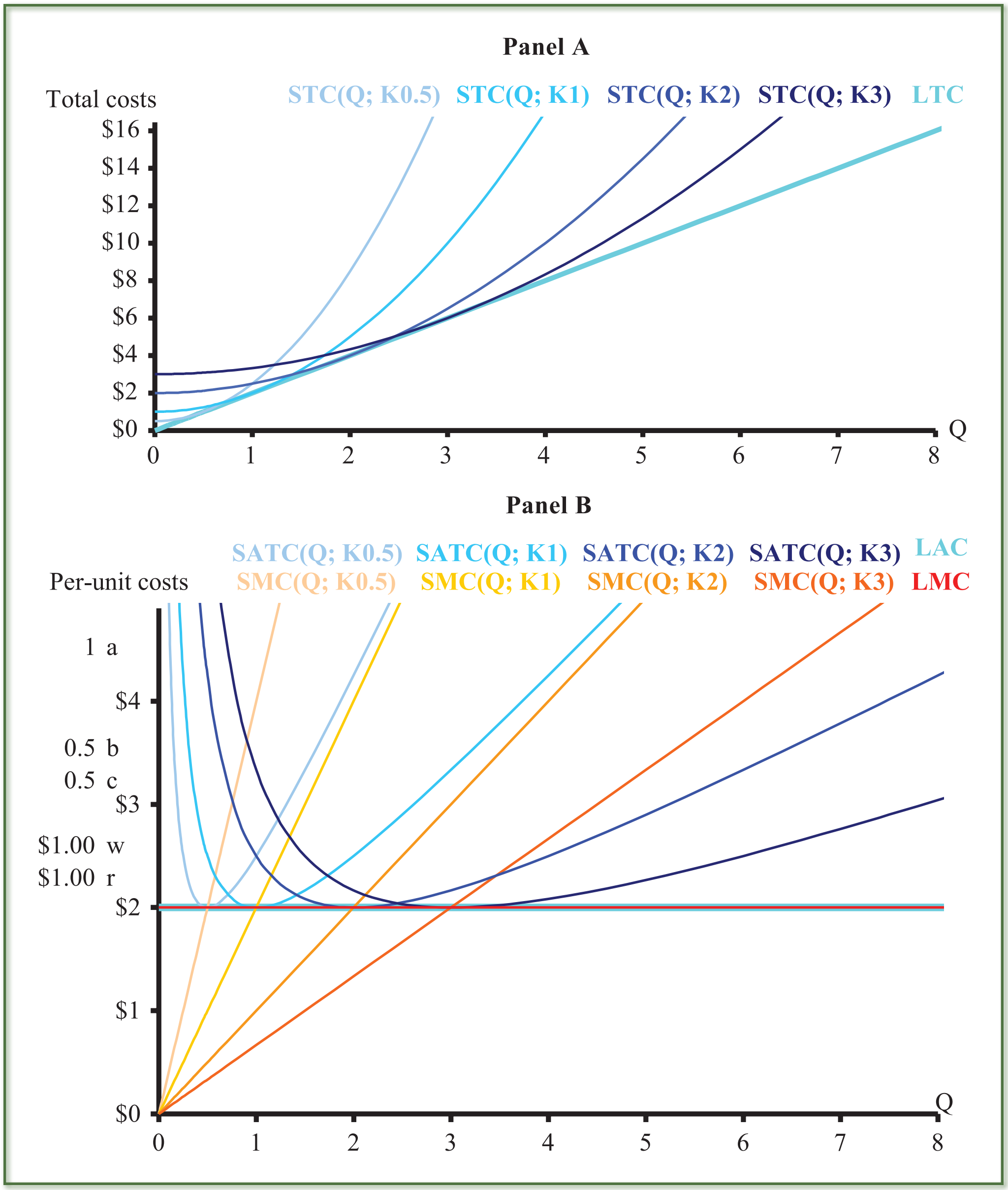


FIGURE 11.18 Cobb-Douglas Cost Function

Constant returns to scale (CRTS): Linear long run, quadratic short run

Similarly, if you fix the vertical axis and pull the right edge to the right, the shape of the curve will not change, but its horizontal placement will change. Finally, if you pull both horizontally and vertically at the same time, the shape does not change, but its horizontal and vertical placement does. For example, the various STC curves in Figure 11.18A are the same shape and can be obtained from each other by horizontal and vertical pulls, while the SATC and SMC curves in Figure 11.18 are also the same shape as each other but require horizontal pulls only.

Focus initially on the long-run cost functions in Equation 11.11. The shapes of the three long-run cost functions are determined entirely by the magnitude of b + c. Basically, the long-run total cost function in Equation 11.11b is a power function of Q, where the exponent attached to Q is 1/(b + c). As discussed in Section 9.4, returns to scale are determined by the magnitude of b + c. When b + c = 1, we have constant returns to scale and consequently linear long-run cost functions, as discussed in Section 11.3. (This is why long-run cost is linear in Equation 11.13a when b = c = 0.5.) We see the same result algebraically in Equations 11.11c and 11.11d (LAC = LMC = H(w, r, *a*, b, c) because

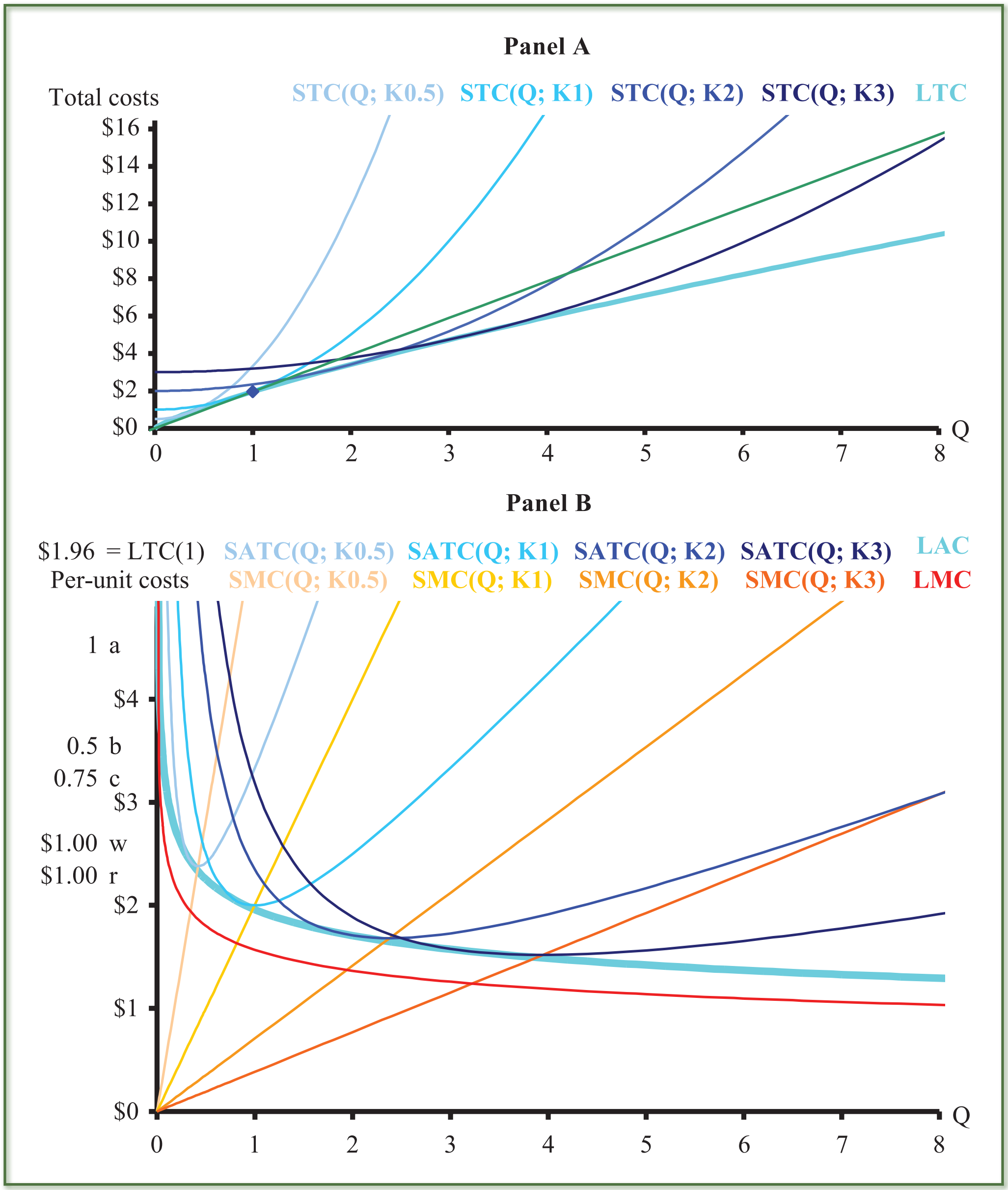


FIGURE 11.19 Cobb-Douglas Cost Function

Increasing returns to scale (IRTS): Convex downward long run, quadratic short run

b + c = 1 implies 1/(b + c) – 1 = 0 and Q^0 = 1 for any Q). For instance, H(w, r, *a*, b, c) = 2, given these values of *a*, b, c, w, and r; therefore, LAC and LMC are flat at a height of 2 in Figure 11.18B.

When b + c > 1, we have increasing returns to scale. An example of increasing returns to scale (IRTS) is shown in Figure 11.19, which differs from Figure 11.18 in only two respects: In Figure 11.19A, c = 0.75, and a green ray has been added to represent LTC(1). The blue diamond at (1, LTC(1)) is on the long-run total cost curve, but it does *not* coincide with that curve unless production exhibits constant returns to scale, as was true in Figure 11.18. In the absence of CRTS, the line is provided to make the curvature of the LTC curve more obvious. In this instance, LTC is convex downward because its shape is a power function, where the power is less than one (1/(b + c) = 4/5 < 1). In Figure 11.19B, both LAC and LMC are declining functions of Q because they are power functions, where the power is less than zero (1/(b + c) – 1 = –1/5 < 0).

The reverse holds true in Figure 11.20A, where c = 0.25 (and b remains at b = 0.5), so the Cobb-Douglas production function exhibits decreasing returns to scale. LTC is

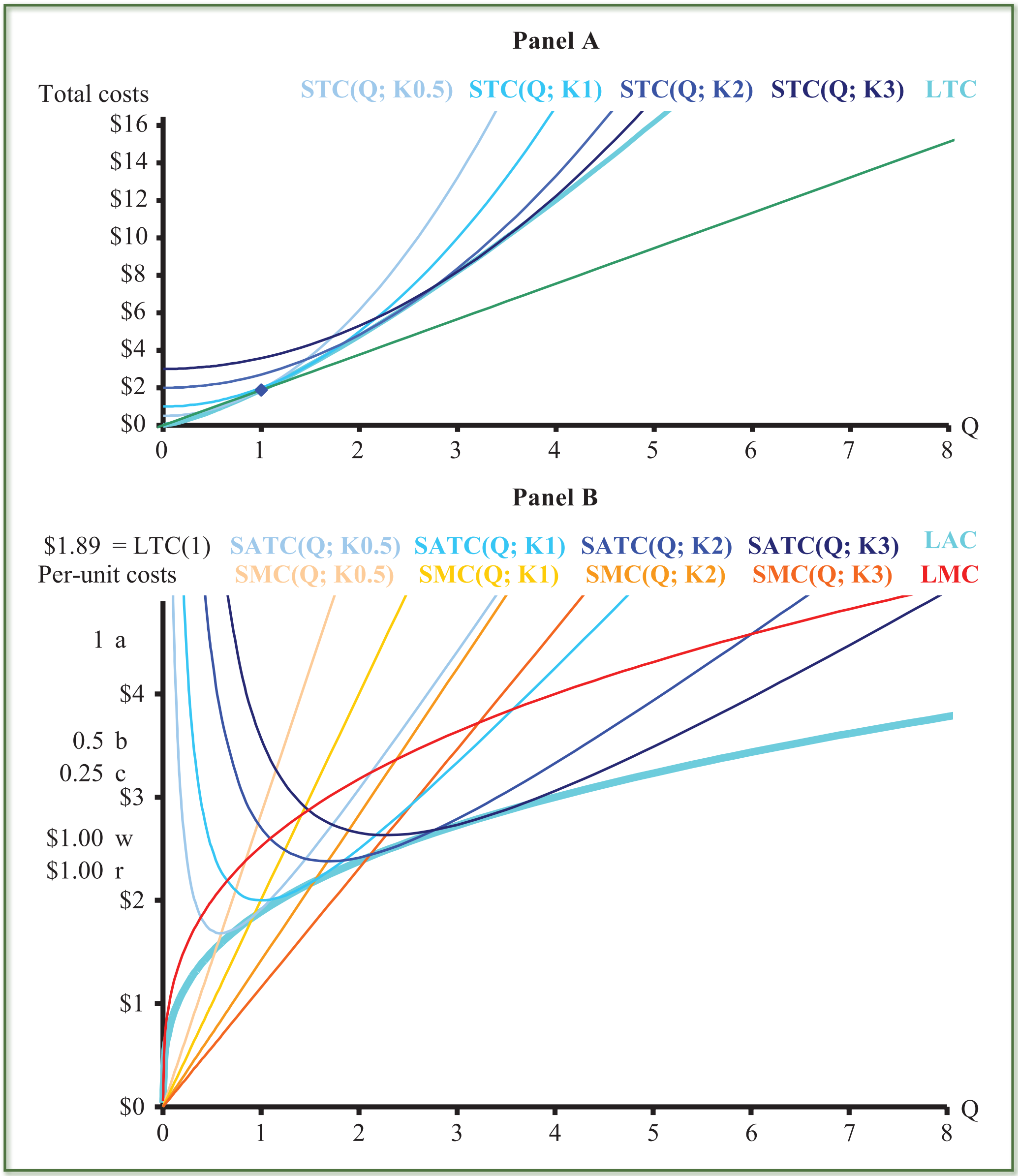


FIGURE 11.20 Cobb-Douglas Cost Function

Decreasing returns to scale (DRTS): Convex upward long run, quadratic short run

convex upward, given decreasing returns to scale (DRTS), because its shape is a power function, where the power is greater than one (1/(b + c) = 4/3 > 1). In Figure 11.20B, LAC and LMC are increasing functions of Q because they are power functions, where the power is greater than zero (1/(b + c) – 1 = 1/3 > 0).

The relative orientation of average and marginal holds in this instance as well: Marginal below average implies average is declining (in Figure 11.19), and marginal above average implies average is increasing (in Figure 11.20). This is confirmed algebraically by a quick examination of Equations 11.11c and 11.11d. The only difference between LAC and LMC is the (b + c) in the denominator of LMC. The two functions have the same shape; they only differ by their relative placement. When b + c < 1, 1/(b + c) > 1, and so LMC > LAC. (Think of LMC as being "stretched" upward relative to LAC in Figure 11.20.) The reverse holds true when b + c > 1, 1/(b + c) < 1, so LMC < LAC. (Think of LAC as being "stretched" upward relative to LMC in Figure 11.19.)

Each of the short-run total cost functions in Figures 11.18–11.20 is quadratic in nature. This may not be obvious in the Panel A's in each figure, but the Panel B's show this in another way: Each short-run marginal cost curve is linear. This will be the case only if total cost is quadratic.[13] The power of the STC power function is determined by the magnitude of b (or more specifically, by 1/b). Each STC function in Figures 11.18–11.20 is quadratic because b = 0.5 in each figure; therefore, SMC and SAVC are linear functions of quantity in Equations 11.12f and 11.12g (since 1/b – 1 = 1 if b = 0.5). Average variable cost has been omitted from these figures, but it differs only in placement from marginal cost because SMC = SAVC/b, using Equations 11.12f and 11.12g. This is analogous to the relation between LAC and LMC discussed in the previous paragraph with one difference. The parameter b must be less than one for the production function to satisfy the law of diminishing marginal product, as discussed in Section 9.3; therefore, 1/b > 1, and hence, SMC > SAVC.[14]

When b ≠ 0.5, the short-run total cost functions will no longer be quadratic, and hence, marginal and average variable cost functions will no longer be linear. To focus attention on the shape of these short-run functions, c is adjusted in Figures 11.21 and 11.22 to maintain CRTS (b + c = 1). If b < 0.5, then 1/b > 2, and 1/b – 1 > 1, which leads the short-run marginal cost curve to be convex upward (the power of the SMC power function is greater than one).

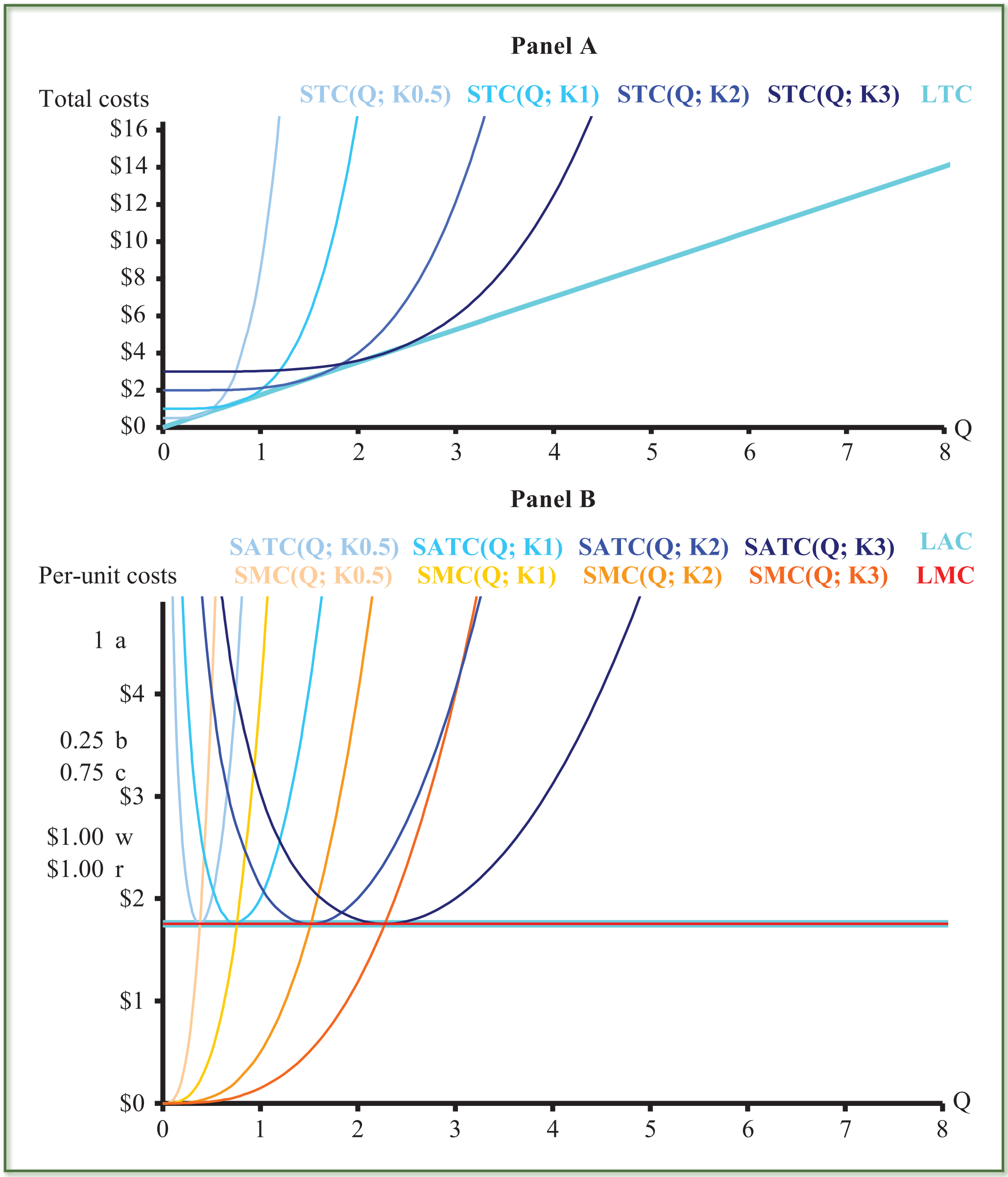


FIGURE 11.21 Cobb-Douglas Cost Function

CRTS: Linear long run, quartic short run

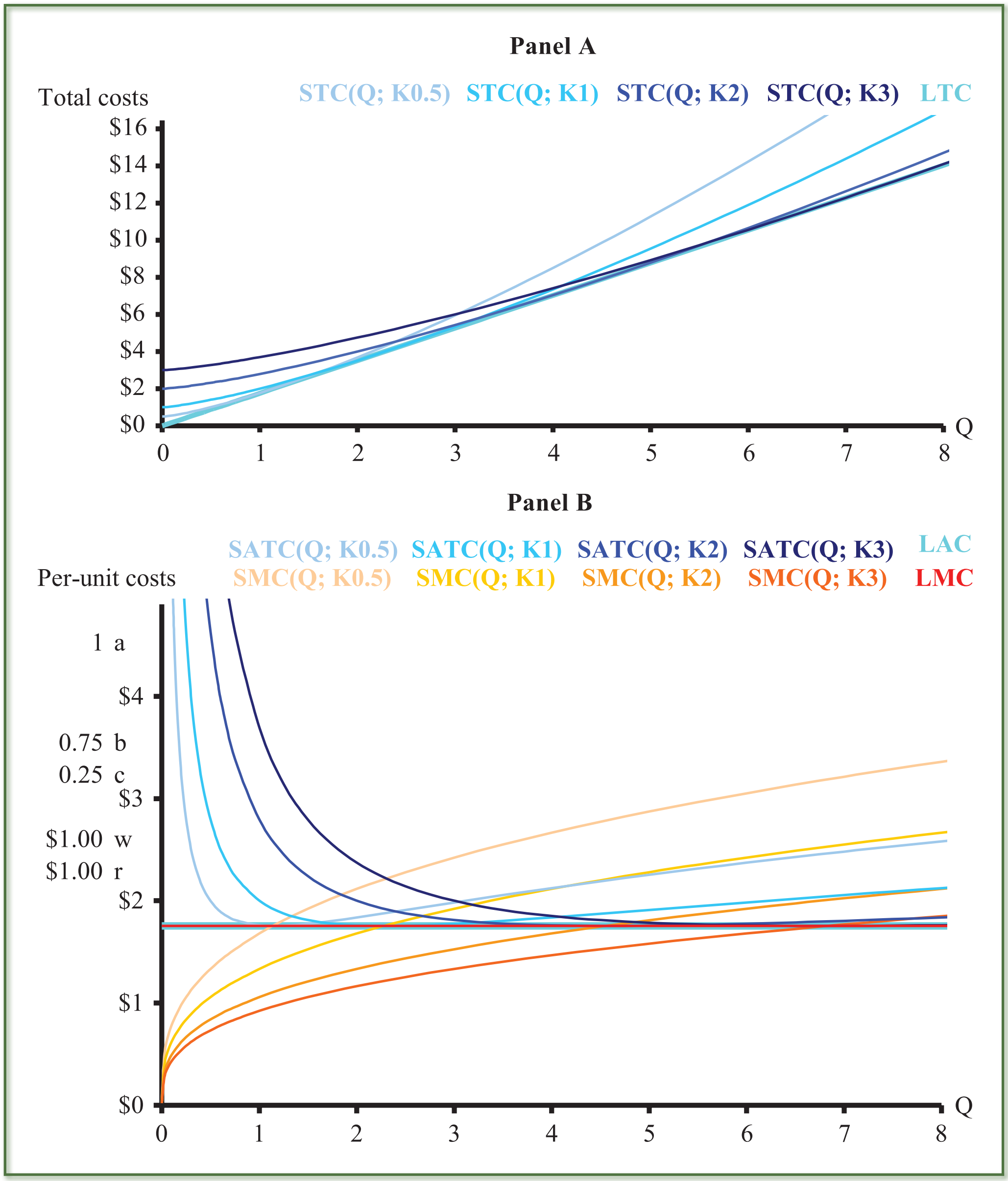


FIGURE 11.22 Cobb-Douglas Cost Function

CRTS: Linear long run; short run is a function of $Q^{4/3}$

This is depicted in Figure 11.21 for b = 0.25. In this instance, 1/b = 4, and STC is a quartic function of Q, and SMC is a cubic function of Q. The convexity of short-run marginal cost reverses if b > 0.5, since 1/b < 2 and 1/b – 1 < 1. STC remains convex upward, since 1/b > 1; STC must be convex upward for a linear LTC curve to be the envelope of STCs, but SMC is now convex downward (since its power is less than one). This is depicted in Figure 11.22 for b = 0.75, so 1/b = 4/3, and STC is a function of Q raised to the 4/3 power. SMC is now a function of $Q^{1/3}$; the cube root function. To examine what happens as b increases (and c decreases to maintain b + c = 1), look from Figure 11.21 to Figure 11.18 to Figure 11.22. For fixed capital, the cost-minimizing output level increases as b increases.

The focus thus far has been on the parameters b and c, the exponents attached to labor and capital in the CD production function. The other parameters have the effect on cost expected. Three parameters remain to be considered: *a*, w, and r. The parameter *a* is a scaling factor in the CD production function; thus far in the discussion, we have set *a* = 1. If *a* increases, then more output can be produced from a given amount of capital and labor.

TABLE 11.2 Cost Curves Based on the Cobb-Douglas Production Function $Q(L, K) = a \cdot L^b \cdot K^c$

<table>
<tr><th colspan="5">Shape and Orientation of Long-Run Cobb-Douglas Cost Curves</th></tr>
<tr><th>Condition</th><th>Figure</th><th>RTS</th><th>Shape of LTC</th><th>LAC and LMC</th></tr>
<tr><td>b + c < 1</td><td>11.20</td><td>DRTS</td><td>Convex upward</td><td>Increasing in Q, LMC > LAC</td></tr>
<tr><td>b + c = 1</td><td>11.18</td><td>CRTS</td><td>Linear</td><td>Constant in Q, LMC = LAC</td></tr>
<tr><td>b + c > 1</td><td>11.19</td><td>IRTS</td><td>Convex downward</td><td>Decreasing in Q, LMC < LAC</td></tr>
<tr><th colspan="5">Shape and Orientation of Short-Run Cobb-Douglas Cost Curves</th></tr>
<tr><th>Condition</th><th>Figure</th><th colspan="2">Shape of SVC and STC</th><th>Shape of SMC and SVC | Orientation</th></tr>
<tr><td>0 < b < 0.5</td><td>11.21</td><td colspan="2">Power function greater than quadratic</td><td>More than linear in Q | SMC > SAVC</td></tr>
<tr><td>b = 0.5</td><td>11.18–11.20</td><td colspan="2">Quadratic cost function</td><td>Linear in Q | SMC > SAVC</td></tr>
<tr><td>0.5 < b < 1</td><td>11.22</td><td colspan="2">Power function less than quadratic</td><td>Less than linear in Q | SMC > SAVC</td></tr>
<tr><th colspan="2">Change Parameter</th><th colspan="3">Effect of an Increase in This Parameter</th></tr>
<tr><td colspan="2">Productivity factor, a</td><td colspan="3">Decreases all short- and long-run cost curves</td></tr>
<tr><td colspan="2">Wage rate, w</td><td colspan="3">Increases SVC, STC, SAVC, SATC, SMC, and all long-run cost curves; does not change SFC or SAFC</td></tr>
<tr><td colspan="2">Capital cost, r</td><td colspan="3">Increases SFC, STC, SAFC, SATC, and all long-run cost curves; does not change SVC, SAVC, or SMC</td></tr>
</table>

Put another way, the cost of producing a fixed amount of output declines. An increase in wage rate increases variable costs of production: Marginal cost rotates upward but has no effect on fixed cost. By contrast, an increase in the rental rate on capital increases fixed cost but has no effect on variable cost. Each of these conclusions is most easily seen by using the sliders in the Excel file for Figures 11.18–11.22. The results described in this section about Cobb-Douglas cost functions are summarized in Table 11.2.

Summary

Cost of production is a direct outgrowth of production technology. The direct linkage between inputs used to produce output and the cost of those inputs is the subject of Chapter 10. The focus there is on the cost-minimizing input mix for producing a given level of output. The connection between input choice and output cost is most directly seen in this chapter in the geometric analysis that begins and ends the chapter. Between these geometric bookends, the analysis examines cost as a function of output. The cost functions derived in this chapter form the basis for profit-maximizing decisions by firms.

Various notions of cost are examined and compared in Section 11.1. Opportunity cost—the cost of a resource in its next best alternative use—forms the basis for economic analysis of cost. Economic cost is distinguished from accounting cost, as are explicit versus implicit cost and fixed versus variable cost. Sunk cost—a cost that has already occurred and that cannot be recovered—is also distinguished as a separate notion of cost.

Cost curves show cost as a function of output. Output is the horizontal axis for cost diagrams, but the vertical axis can be one of two things: total dollars spent or dollars spent per unit of output. The two versions of cost curves are, of course, related, and that relation forms the basis for much of the analysis in the chapter.

Short-run total cost is the sum of fixed and variable cost. Dividing each of these three cost functions by quantity gives us their per-unit counterparts: average total cost, average fixed cost, and average variable cost. Marginal cost, the final short-run per-unit cost function, is, arguably, the most important, as we will soon see. Marginal cost is the incremental cost of production. How much does an extra unit of output cost? The slope of the total cost or the variable cost function provides an answer to this question.

Marginal cost has the same relation to each of the average cost curves that marginal product has to average product in Chapter 9. When marginal is below average, average is declining, and when marginal is above average, average is increasing. Marginal cost equals average cost at minimum average cost. This is true for both average variable cost and for average total cost.

Often, the short-run total cost curve is initially convex downward, due to an initial range of increasing marginal productivity for variable factors of production. The convexity eventually changes, and the cost curve becomes convex upward, due to the law of diminishing marginal product. Total cost curves that have this upward-sloping "S" shape produce U-shaped average variable, average total, and marginal cost curves. Although it is not necessary to model cost curves using cubic equations, this is the simplest functional form that produces these results.

Long-run costs have no fixed component because, in the long run, there are no fixed factors of production. Therefore, there are only three long-run cost curves, as opposed to seven short-run cost curves. Long-run total cost is the sole long-run total cost curve, and long-run average and long-run marginal cost are per-unit cost curves.

In the long run, firms have extra options available that they do not have in the short run; for example, they can change plant size in response to market signals. As a result, long-run costs are always less than or equal to short-run costs. Indeed, one way to conceptualize long-run costs is that the long run is the best (lowest cost) of each possible short run. Formally, long-run total cost is the envelope of short-run total cost curves, and long-run average cost is the envelope of short-run average total cost. The same is not true for long-run marginal cost.

Long-run cost functions based on constant returns to scale production technologies have linear long-run total cost and flat long-run average and marginal cost. Another common shape for long-run total cost is similar to the upward-sloping "S" shape of a short-run variable cost function. The interpretation of these two visually similar functions is, however, entirely different. Short-run variable cost has this shape due to increasing and then decreasing marginal productivity of variable factors of production. By contrast, the long-run total cost curve has this shape due to initial economies of scale in the production process, which are eventually replaced by diseconomies of large-scale production.

The chapter concludes with an analysis of cost curves based on Cobb-Douglas production technology. As noted in Chapter 9, Cobb-Douglas production functions do not have varying returns to scale, and factors of production have diminishing marginal product over the entire range of usage of the factors of production. Therefore, short-run total cost curves deriving from Cobb-Douglas technology are convex upward, and long-run total cost curves may be convex upward (if the production process exhibits decreasing returns to scale), linear (if the production process exhibits constant returns to scale), or upward sloping but convex downward (if the production process exhibits increasing returns to scale). But the Cobb-Douglas production function will not have varying returns to scale without further modification of the function.

Review Questions

Define the following terms:

opportunity cost	total cost (TC)	average fixed cost (AFC)
economic cost	sunk cost	long-run total cost curve, LTC(Q)
explicit cost	options contract	long-run marginal cost curve, LMC(Q)
implicit cost	per-unit costs	long-run average cost curve, LAC(Q)
accounting cost	marginal cost (MC)	the geometric concept of an *envelope*
fixed cost (FC)	average total cost (ATC)	
variable cost (VC)	average variable cost (AVC)	

Match the following phrases:

a. Average fixed cost plus average variable cost	1. $\Delta VC(Q)/\Delta Q$
b. Fixed cost	2. $VC(Q) + FC(Q)$
c. Explicit cost plus implicit cost	3. Minimum average total cost
d. Marginal cost equals average variable cost	4. $Q \cdot (ATC(Q) - AVC(Q))$
e. Average fixed cost at output level Q	5. Economic cost
f. Total cost at output level Q	6. $ATC(Q) - AVC(Q)$
g. Marginal cost	7. Minimum average variable cost
h. Marginal cost equals average total cost	8. $ATC(Q)$

Provide short answers to the following:

a. If at a given output level, you know that marginal cost is more than average variable cost, what happens to average variable cost as output expands by a small amount?

b. If at a given output level, you know that marginal cost is less than average total cost, what happens to average total cost as output expands by a small amount?

c. Why are there seven short-run cost curves but only three long-run cost curves?
d. If average variable cost is declining at a given output level, is it possible that average total cost is increasing at that same output level?
e. If average variable cost is increasing at a given output level, is it possible that average total cost is declining at that same output level?
f. If both average variable cost and average total cost are declining at a given output level, what is true about marginal cost at that output level? (Must it also be declining?)
g. If both average variable cost and average total cost are increasing at a given output level, what is true about marginal cost at that output level? (Must it also be increasing?)
h. What is the geometric relation between short-run total cost curves for various plant sizes and the long-run total cost curve for this firm?
i. What is the geometric relation between short-run average total cost curves for various plant sizes and the long-run average cost curve for this firm?

Check your answers to the *matching* and *short-answer* exercises in the Answers Appendix at the end of the book.

Notes

1. There are excellent educational materials on the Chicago Board Options Exchange website. Go to http://www.cboe.com/

2. Had the example been of an "eat-in" pizza shop, the production function would no longer be Leontief because eat-in pizzas do not require a box. Of course, an "eat-in" shop would have been more complex to examine, due to the extra costs involved in such an operation.

3. As with marginal utility and marginal product, marginal cost is more formally defined using derivatives. In this instance, MC(Q) = dTC(Q)/dQ = dVC(Q)/dQ. These two derivatives are equal because TC and VC only differ by a constant, FC, and the derivative of a constant is zero.

4. You have seen an analogous piece of geometry in the consumer context when we discussed quasilinear preferences. The defining geometric feature of quasilinear preferences was that indifference curves were vertical translates of each other. As a result, the slope of the indifference curve (MRS) depended only on how much x was being consumed (rather than how much x *and* y was consumed). The same result holds here: The slopes of TC(Q) and VC(Q) are the same for any level of output and only depend on the level of Q under consideration.

5. If only discrete plant sizes were possible, then the production function would have more of the look of the discrete goods utility function described in Section 5.6. Suppose, for example, that capital is discrete and labor is continuous. Then the production function would be defined on horizontal lines in (L, K) space, where each horizontal line is an available plant size.

6.The K_i numbers do not represent cardinal differences between capital stock. K4 is not twice the size of K2, and K16 is not four times the size of K4. This is also seen in Figure 11.9. We knew that the small plant produced Q = 4 at minimum cost, and the large plant produced Q = 20 at the same minimum average total cost, despite FC_{small} = \$32 not being one-fifth the size of FC_{large} = \$400.

7. Exact values for the various cost functions require the actual cost functions. The algebraic functions are unimportant here; the figures allow you to see these as approximate values.

8. To understand why this is true requires a basic understanding of derivatives. This understanding is not necessary to do well in intermediate microeconomics (just as it was unnecessary to explain why the marginal utilities in Table 4.1 were what they were).

9. The proof in this instance requires calculus. Set dMC(Q)/dQ = 0 and solve for Q.

10. This is a perfect place to use Excel's **Goal Seek** function, discussed in Section 7.5. If you put labels in A1:D1 and a sample value of Q in cell A2, and the ATC and MC equations in B2 and C2, then you can place the difference in D2 and **Goal Seek** by setting D2 to zero by changing A2. An example of this is provided in Table 11.1. The Chapter 11 Appendix discusses graphing cubic cost functions using Excel.

11. As noted in the Mathematical Appendix (available in the online version of this text), the quadratic formula states that the equation $A \cdot x^2 + B \cdot x + C = 0$ has solutions $x = (-B \pm (B^2 - 4 \cdot A \cdot C)^{0.5})/(2 \cdot A)$. In Equation 11.6g, $A = 3 \cdot d$, $B = -2 \cdot c$, and $C = b$. Two solutions occur when the part under the square root (the part raised to the 0.5 power) is positive; there is one solution when the square root is zero. To have no solution, the part under the square root must be negative, a condition that simplifies to Equation 11.10.

12. As above, marginal cost is the derivative of total cost. If you know calculus make sure you can derive this yourself. If not, do not worry; marginal cost will be given for any total cost function used.

13. The proof of this assertion is simple if you know calculus. A derivative reduces the power of the function by one, so if you end up with a function to the first power (linear), then you must have started with a function that was to the second power.

14. We need not focus on the functional form of the SATC function. This function will necessarily include a 1/Q component due to the inclusion of fixed cost in the short run. For large values of output, SAFC becomes negligible, and SATC closely tracks SAVC. Indeed, mathematicians say that SATC is asymptotic to SAVC for large Q; as Q approaches zero, SATC is asymptotic to the vertical axis.

Appendix:

Using Excel to Graph Cubic Cost Functions

This appendix is most effectively read if you have the Excel file for this appendix open as you read and if you enter equations described into the cells suggested on the **DoItYourself** worksheet. The cubic cost curves derived in Section 11.4 are readily graphed, using Excel. This appendix focuses on the per-unit cost curves derived in that section, but the methods discussed here could just as easily produce a total cost graph. They also could be used to provide graphs of the Cobb-Douglas cost functions discussed in Section 11.5. This appendix assumes some familiarity with basic Excel functions, such as using absolute versus relative referencing, dragging equations, and using **Goal Seek**. (These topics are discussed in the Mathematical Appendix [available in the online version of this text] and in Chapter 19, but they are considered here as well.)

The discussion in this appendix relates to the Excel spreadsheet shown in Table 11A.1. Panel A in Table 11A.1 provides the actual Excel spreadsheet, and Panel B provides equations for seven "critical cells" (highlighted in yellow). If care is taken in entering the equations in these cells, then the equations can be dragged to create the entire spreadsheet. (Note that column widths in Panel B differ from Panel A to accommodate the size of each equation.) The blue cells have equations entered into them via dragging from above. Finally, the six green cells represent the data used to produce the graphs. Panel A includes two graphs in E1:I21. The first graph depicts the raw output, and the second provides a more finished presentation.

Consider the total cost function:

$$TC(Q) = 9800 + 760 \cdot Q - 12 \cdot Q^2 + 0.1 \cdot Q^3. \quad \textbf{(11A.1)}$$

Rather than typing numbers into equations (as you would do, for example, if you were entering Equations 11.8a–11.8d), it is more effective to place the cubic total cost parameters *a*, b, c, and d from Equation 11.6a in a data area at the top of the file and then use those parameters in building the equations (Equation 11.6a is reproduced in cell C1:D1). Place the values for *a*–d in cells A1:A4.

It is worthwhile to build a bit of flexibility into the file. If you are not familiar with the cost function under consideration, for example, it is worth having a parameter that provides the change in quantity between Q values so that the size of the graph can be easily adjusted (cell A5). The size of the quantity runner (in A8:A18) is readily adjusted in this instance by changing A5. We use dQ = 10 in the present instance because each of the minimums of MC, AVC, and ATC are captured over the range 0–100.

To build the quantity runner in this instance, simply start at Q = 0 (type the number 0 in cell A8). In cell A9, type the equation =A8+$A$5 (the $ signs fix the reference). Drag this from A9 to A18 by moving to the lower right corner of the cell and press down on the mouse once the crossed arrows turn into a **+** sign; then drag to A18. The number 100 will appear in cell A18. Click on cell A18, and the equation in that cell should be =A17+$A$5 (the lack of a dollar sign in the first term in the equation means that A8 has turned into A17, but A5 has remained fixed because cell A9 has been dragged to cell A18).

The per-unit cost equations can be entered using one equation for each per-unit cost function in cells B8, C8, and D9. Equations 11.6g, 11.6f, and 11.6d in the text are noted in cells B6:D6 for ease of reference. For example, marginal cost from Equation 11.6g is:

$$MC(Q) = b - 2 \cdot c \cdot Q + 3 \cdot d \cdot Q^2. \quad \textbf{(11A.2a)}$$

This is entered in cell B8 as:

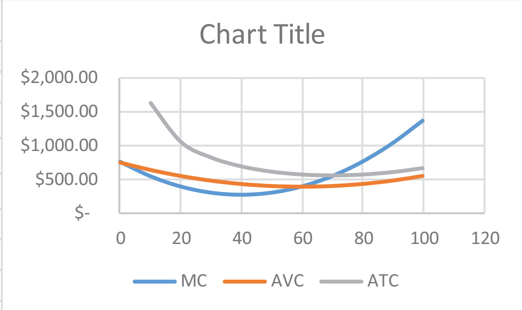
=$A$2-2*$A$3*A8+3*$A$4*A8^2 **(11A.2b)**

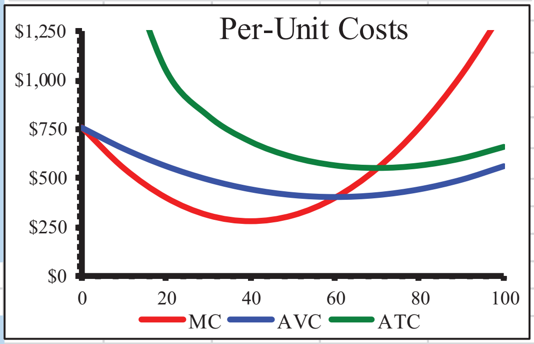
Notice that each equation uses $ signs in front of the letter and number reference to each of the parameters b–d (A2:A4), but no $ signs in front of the letter or number reference to Q (A8). This means that when this equation is dragged to B18, the quantity values will change, but the coefficient values for b–d will remain fixed. Notice also that you

TABLE 11A.1 Cubic Per-Unit Costs in Excel

Panel A: Spreadsheet for cubic per-unit costs (see Panel B for formula key)

9800	a = FC	TC = a + bQ - cQ^2 + dQ^3			
760	b = linear term				
12	c = quadratic term				
0.1	d = cubic term				
10	dQ				
Eqs.11.6:	b-2cQ+3dQ^2	b-cQ+dQ^2	a/Q+AVC		
Q	MC	AVC	ATC		
0	$ 760.00	$ 760.00			
10	$ 550.00	$ 650.00	$ 1,630.00		
20	$ 400.00	$ 560.00	$ 1,050.00		
30	$ 310.00	$ 490.00	$ 816.67		
40	$ 280.00	$ 440.00	$ 685.00		
50	$ 310.00	$ 410.00	$ 606.00		
60	$ 400.00	$ 400.00	$ 563.33		
70	$ 550.00	$ 410.00	$ 550.00		
80	$ 760.00	$ 440.00	$ 562.50		
90	$ 1,030.00	$ 490.00	$ 598.89		
100	$ 1,360.00	$ 560.00	$ 658.00		
	Minimums				
Eq.11.7	MC	AVC	ATC		
40	$ 280.00	$ 440.00	$ 685.00		
60	$ 400.00	$ 400.00	$ 563.33	MC-ATC	
70	$ 550.00	$ 410.00	$ 550.00	-	**Goal Seek** E23 to zero by changing A23





Panel B: Formulas for the (yellow) critical cells in Panel A

	A	B	C	D	E
1	9800	a = FC	Eq. 11.6a. TC(Q) = a + bQ - cQ^2 + dQ^3		# of cells
2	760	b = linear term	Green cells require data entry.		6
3	12	c = quadratic term	Yellow cells require inputting critical equations.		7
4	0.1	d = cubic term	Blue cells with borders require dragging critical equa		47
5	10	Q (size of the quantity runner)			
6	Eqs.11.6g,f,d:	MC(Q) = b-2cQ+3dQ^2	AVC(Q) = b-cQ+dQ^2	ATC(Q)=a/Q+AVC(Q)	
7	Q	MC	AVC	ATC	
8	0	=$A$2-2*$A$3*A8+3*$A$4*A8^2	=$A$2-$A$3*A8+$A$4*A8^2		
9	=A8+$A$5			=$A$1/A9+C9	
10					
11					
12					
13					
14					
15					
16					
17					
18					
19		Minimums			
20	Eq.11.7	MC	AVC	ATC	
21	=A3/(3*A4)				
22	=A3/(2*A4)				MC-ATC
23	100	**Goal Seek** MC-ATC (E23) to zero by changing Q (A23) to obtain minimum ATC.			=B23-D23

should not add spaces when entering equations in Excel. (If you are using the Excel file and click on this cell on the main worksheet, this equation is also shown in the **Formula Bar** [above the C|D column header to the right of ***fx***], since this shows the equation in the active cell.)

It is worth remembering that the minus sign in front of the linear term in marginal cost is because total cost was defined with a minus sign in front of the quadratic term $c \cdot Q^2$. This is simply the convention chosen for the text. Had we assumed a plus sign in front of this term, then c in cell A3 would have to be entered with a minus sign. Both conventions would have produced the same result (but the equations in B8 and C8 would have used a plus sign in front of the linear term [and the equations in cells A21 and A22 would both have led with a minus sign]).

The other two per-unit cost curves are entered in the same way. The specific equations for AVC in cell C8 and ATC in D9 are shown in those respective cells in Panel B and are reproduced here:

$$AVC(Q) = b - c \cdot Q + d \cdot Q^2. \quad \textbf{(11A.2c)}$$

This is entered in cell C8 as:

=$A$2-*$A$3*A8+*$A$4*A8^2 **(11A.2d)**

$$ATC(Q) = a/Q + AVC(Q). \quad \textbf{(11A.2e)}$$

ATC is not defined at Q = 0; therefore, we start at the first nonzero Q value in cell D9 as:

=$A$1/A9+C9 **(11A.2f)**

Once these equations are entered in C8 and D9 (making sure $ signs are in the correct positions), the equations can be dragged to C18 and D18.

The graph is readily obtained from the cost information that has just been created:

If using Excel 2007, highlight A7:D18 and then click the **Insert** ribbon, click **Scatter**, then click **Scatter with Smooth Lines**. The upper graph will appear.

If using Excel 2003, highlight A7:D18 and then click the **Chart Wizard** icon on the **Standard** toolbar. The Chart Wizard popup menu will appear. Click **XY (scatter)** from the Chart type menu and **Scatter with data points connected by smoothed lines without markers** from the Chart subtype menu. Click **Finish**, and the upper graph will appear.

The lower graph simply cleans up the preliminary version by adjusting axes, changing the color and width of curves, and adjusting the background and borders of the graph. Each of these tasks is easily accomplished by clicking on various parts of the graph.

The final part of Table 11A.1, in rows 21–23, obtains minimum values of each function. As noted in the text, two of these values are readily derived from the parameters of the total cost function. In particular, marginal cost is minimized when $Q = c/(3 \cdot d)$, and average variable cost is minimized when $Q = c/(2 \cdot d)$, according to Equations 11.7a and 11.7b. These equations are entered into cells A21 and A22, using the coefficients for c and d from cells A3 and A4. MC, AVC, and ATC are copied from an earlier row and pasted into cells B21:D21 and then dragged to B23:D23 to obtain values of each function at these output levels. Note that MC = AVC at minimum AVC, according to cells B22 and C22 (as expected).

This same strategy is used to find minimum ATC, as discussed in Table 11.1 in the chapter. A number is placed in cell A23 (since **Goal Seek** cannot work on an equation). The actual value is not important, as it simply provides a starting point for Excel's search process. (Recall that you reach **Goal Seek** by clicking **Data, What If Analysis, Goal Seek** in Excel 2007, or **Tools, Goal Seek** in Excel 2003.) The difference between ATC and MC is calculated in cell E23. Minimum ATC is found by Goal Seeking on cell E23 to zero by changing A23. The result is shown in the table: As expected, ATC = MC = $550 when Q = 70. The minimum value for each of the three curves is visually confirmed on the graph (but the quantity and dollar values cannot be seen with the same degree of accuracy in a graph). In this instance, the minimum MC, AVC, and ATC answers derived in rows 21–23 are also seen in rows 12, 14, and 15 because Q = 40, 60, and 70 in those rows.